# METERING WATER FLOW EXITING OPEN-ENDED PIPES

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## Introduction

Metering water flows in pipes is important in a wide range of applications. A special case is where water exits from an open-ended pipe to the atmosphere. Such applications are found in agricultural irrigation and drilling for fresh water and low-temperature geothermal water, for example. The need for a simple and inexpensive metering method may arise where the installation of accurate metering is not practical and/or too expensive. Relationships for vertical and horizontal pipes will be derived and presented in the following text.

## Vertical Pipes

Consider a vertical pipe or wellhead where water exists to the atmosphere as illustrated in enclosed figure. Bernoulli's equation for the water flow before (=1) and after (=2) exiting can be written

$$p_1 + \frac{1}{2}\rho u_1^{\,2} + \rho\, g\, h_1 = p_2 + \frac{1}{2}\rho u_2^2 + \rho\, g\, h_2$$

where p (Pa) is pressure, ρ (kg/m$^3$) water density, g (m/s$^2$) the gravitational constant and h (m) the absolute height. The pressure immediately before and after the pipe exit will be close to atmospheric pressure ($p_1$=$p_2$) and can therefore be cancelled out from the equation. At the top of the water column (height $h_2$) the water velocity is zero ($u_2$=0). What remains in the equation is the following

$$\frac{1}{2}\rho u^2 + \rho\, g\, h_1 = \rho\, g\, h_2$$

Cancelling out water density and rearranging gives

$$\frac{1}{2}u_1^2 = g\left(h_2 - h_1\right)$$

Furthermore, expressing the height of the water column as a difference (Δh) the following relationship results

$$u_1 = \sqrt{2\, g\, \Delta h}$$

Interestingly, this relationship is the same as Torricelli's Law for the flow velocity through a hole in a tank with a constant liquid level.

The subscript 1, indicating the water velocity exiting the pipe, can now be dropped without any loss of clarity. And since the flowrate q ($m^3/s$) is given by the relationship

$$q = u\,A$$

where A ($m^2$) is the cross-sectional flow area, the equation becomes

$$q = A\sqrt{2\,g\,\Delta h}$$

This equation is theoretical and does not take into account whatever irreversible losses that arise when water exits and rises from an open-ended pipe. The usual way to take such losses into account is to introduce a discharge coefficient $C_d$ whose value need to be determined from experiments (Gudmundsson 1995). In the present work the value of the discharge coefficient is assumed to be closed to 0.9. This compares to 0.6 for orifice meters, 0.95 for nozzle meters and 0.97 for venturi meters.

$$q = C_d\,A\sqrt{2\,g\,\Delta h}$$

Based on the area equation and given values

$$C_d = 0.9$$
$$A = \frac{\pi\,d^2}{4}\,(m^2)$$
$$g = 9.81\,(m/s^2)$$

the following **practical relationship** results

$$\boxed{q = 3.13\,d^2\,\sqrt{\Delta h}}$$

Based on general considerations it seems reasonable to assume that this practical relationship will give less accurate flowrate value when the height of the water column is less than the pipe diameter. Therefore, the practical relationship should only be relied upon when

$$\Delta h > d$$

This limitation in practical situations can be resolved by installing a smaller diameter pipe on top of the too-large a diameter pipe. That is, the pipe diameter can be adjusted to a smaller diameter pipe to obtain a high-enough water column for more accurate metering.

As an example calculation, for a pipe/wellhead 6 inches (1 inch = 2.54 cm) in internal diameter (ID), water rising vertically 40 cm (0.4 m), the flowrate is approximately

$$q(m^3/s) = 3.13 \times (6 \times 0.0254)^2 \times \sqrt{0.4} = 0.046\,(m^3/s) = 46\,(L/s)$$

The above practical relationship is similar to work presented by Bos (1976/1989). Based on experimental work, Bos (1976/1989) arrived at the following semi-empirical relationship

$$\boxed{q = 3.15 d^{1.99}\,\Delta h^{0.53}}$$

most accurate for

$$\Delta h > 1.4\,d$$

For the same example as illustrated above, the flowrate becomes

$$q(m^3/s) = 3.15 \times (6 \times 0.0254)^{1.99} \times \Delta h^{0.53} = 0.046(m^3/s) = 46(L/s)$$

which gives the same result. Therefore, both the Bos-equation and the simpler (based on square and square-root calculation) practical relationship derived above can be used to calculate the flowrate of water issuing/exiting from an open-ended pipe.

Bos (1976/1989) presented also a semi-empirical equation for small flowrates as follows

$$q = 5.47\,d^{1.25}\,\Delta h^{1.35}$$

most accurate for

$$\Delta h < 0.4\,d$$

For Δh values between 0.4 and 1.4, Bos (1976/1989) recommends using the average of both equations.

**Horizontal Pipes**

Consider a horizontal pipe or wellhead where water exits to the atmosphere as illustrated in enclosed figure. One of the methods found in the literature is the trajectory method (Bos 1976/1989). The water travels a horizontal distance x (m) and falls a vertical distance y (m). The horizontal distance is given by the simple relation

$$x = u_o t$$

where $u_o$ is the velocity where x=0. In the same time t, the water will fall the vertical distance given by the simple relation

$$y = \frac{1}{2} g t^2$$

We note that g×t (m/s) is the vertical downward velocity of the water due to gravity. The water starts at zero and accelerates to subsequent values of g×t (m/s). Therefore, the factor ½ needs to be used in the above relation, to give the average downward velocity of the water.

Since

$$x^2 = u_o^2 t^2$$

the time can be eliminated from the two basic equations, such that

$$\frac{x^2}{y} = \frac{u_o^2 t^2}{\frac{1}{2} g t^2} = \frac{u_o^2}{\frac{1}{2} g}$$

Assuming that the horizontal velocity of the water remains constant for the distances of interest, the subscript o can be dropped without any loss of clarity. Using the physical relation between velocity u (m/s) and flowrate q ($m^3$/s) the following expression results

$$q = A\sqrt{\frac{g}{2}\frac{x^2}{y}}$$

This equation is theoretical and does not take into account whatever losses/gains that arise when water exits horizontally from and open-ended pipe. The distance x (m) is based on a constant water velocity from the pipe-end and into the surrounding air; the trajectory of the issuing water. It seems reasonable to assume that the water velocity decreases with distance away from the pipe-end. This means that the real x (m) value will be smaller than the theoretical x (m) value. Similarly, the real vertical distance y (m) will be larger than the theoretical distance. Taking both of these real distances into account means that the real flowrate q ($m^3$/s) will be larger than the theoretical value. Adding a discharge coefficient $C_d$ to the theoretical expression makes it possible to adjust the theoretical value to the actual value

$$q = C_d A\sqrt{\frac{g}{2}\frac{x^2}{y}}$$

The $C_d$-value needs to be larger than unity. The $C_d$-value for vertical flow above was taken to 10% lower than theoretical (=0.9). It seems reasonable to assume that the $C_d$-value for horizontal flow be taken to be 10% higher than theoretical (=1.1). Charts presented by Bos (1976/1989) suggest that such a value may indeed be correct.

Based on the area equation and given values

$$C_d = 1.1$$
$$A = \frac{\pi d^2}{4}\,(m^2)$$
$$g = 9.81\,(m/s^2)$$

the following **practical relationship** results

$$\boxed{q = 1.91 d^2\sqrt{\frac{x^2}{y}}}$$

As an example calculation, for a pipe 4 inches (1 inch = 2.54 cm) in internal diameter (ID) and water horizontal trajectory of 15 cm (x=0.15 m) and vertical trajectory of 6 cm (y=0.06 m), the flowrate is approximately

$$q(m^3/s) = 1.91 \times (4 \times 0.0254)^2 \times \sqrt{\frac{(0.15)^2}{0.06}} = 0.012(m^3/s) = 12(L/s)$$

In the use of the above equation for horizontal flow of water exiting a pipe, care should be taken to make sure the pipe is truly horizontal. The horizontal section should be as long as possible and not shorter than about 10 pipe diameters. If the pipe has a slight downward angle the calculated flowrate will be too low. Similarly, if the pipe has a slight upward angle the calculated flowrate will be too high. The distances x (m) and y (m) are referenced to the internal diameter of the pipe.

## References


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