NORWEGIAN UNIVERSITY OF SCIENCE AND TECHNOLOGY
Department of Petroleum Engineering and Applied Geophysics

Lecture Notes

OIL AND GAS DELIVERABILITY

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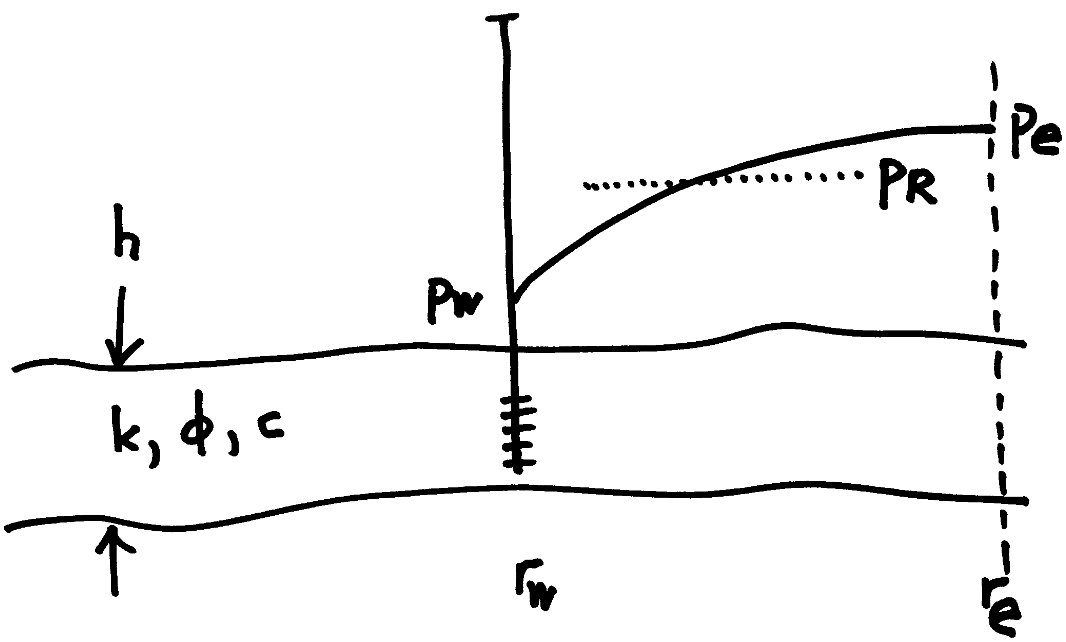

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Introduction

Petroleum engineering is traditionally divided into drilling engineering, production engineering and reservoir engineering. A certain overlap exists between production and reservoir engineering. At universities, pressure transient testing and formation evaluation (logging of wells) can be organised separate or within reservoir engineering. Petroleum engineering is not a very large profession and is not taught at many universities around the world.

In Norway the word petroleum means both oil and gas. Around the world, university departments are sometimes called Petroleum and Natural Gas Engineering. Traditional oil companies are called Exploration and Production companies and the industry thereafter E&P, hence the U&P (utforskning og produksjon) in Norwegian. At NTNU in Trondheim, petroleum engineering and exploration are organized together in one department. The exploration part consists of applied geophysics, primarily seismic, and petroleum geology.

Unlike other engineering disciplines (civil, mechanical, electrical, chemical), petroleum engineering serves a particular industry, the E&P industry. This explains the relative small size of the profession and at the same time the strong applied nature of petroleum engineering. However, due to the importance of the E&P industry in Norway, petroleum engineering education at NTNU and the University of Stavanger are relatively large on a world-wide basis. The Society of Petroleum Engineers (SPE) has bout 88,000 members world-wide.

Petroleum engineering education at NTNU includes all the classical subjects: flow in porous media, phase behaviour, well testing, well logging, reservoir simulation, processing of petroleum, well performance, well completion, drilling equipment, drilling fluids and rock mechanics. Subsea technology is also included at NTNU. Research and development activities span the central subjects of petroleum engineering and a multitude of specialized topics. A recent addition is R&D within Integrated Operations.

Outsiders to petroleum engineering are usually just interested in how much (flowrate) a particular reservoir and wells can produce in terms of oil, gas and water, with what composition, temperature and pressure, and for how long. Such data are then used in the design of platforms, subsea equipment and receiving terminals. What differentiates petroleum engineering from most other classical engineering disciplines is the large degree of uncertainty and variability in production with time; from first oil to abandonment.

A better understanding the fundamentals of oil and gas production will benefit the engineering professions involved in the design and construction of facilities and infrastructure in the E&P industry. There are many ways to present the fundamentals of oil and gas production. One way is to use the concept of deliverability.

The present Lecture Notes stem originally from a part of a course at NTNU for non-petroleum engineers. The bulk of the material was presented with a focus on production engineering issues. The text was later adjusted and extended for use in presenting reservoir and production engineering to students of geothermal reservoirs, in particular the inclusion of pressure transient testing (here as Appendix A). The present version has been adjusted for engineers at Aker Solutions involved in process engineering, to supplement two presentations with the title RESERVOIR TO WELLHEAD: Deliverability in oil and gas production.

Concept of deliverability

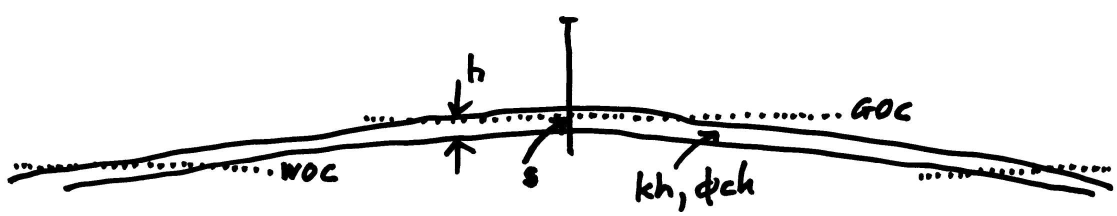
- Analyze individual performances and then synthesize to get deliverability
- Methodology to estimate capacity of wells and field to deliver fluids
- Rate, q, and pressure, p, main parameters
- Reservoir performance, reservoir pressure against cumulative production
- Inflow performance, downhole pressure against flowrate
- Outflow performance, tubing performance, down hole pressure at fixed wellhead pressure (several curves for different wellhead pressures)

Deliverability and Performances
Pressure Profile from Reservoir to Wellhead

- Reservoir performance gives reservoir pressure against cumulative production (or average reservoir pressure with time). For example, p/z method for simple gas reservoirs.
- Inflow performance gives downhole flowing pressure against well production rate. Practically equal to reservoir pressure when no production (well shut-in).
- Tubing performance gives the pressure drop from downhole to wellhead. Assuming a wellhead pressure, the downhole pressure can be calculated (compressible flow in pipes) for different well production rates. One curve for each wellhead pressure.
- Deliverability is the overall effect of the reservoir, inflow and tubing performances. Nodal Analysis is another term use in the industry for Deliverability as used here.

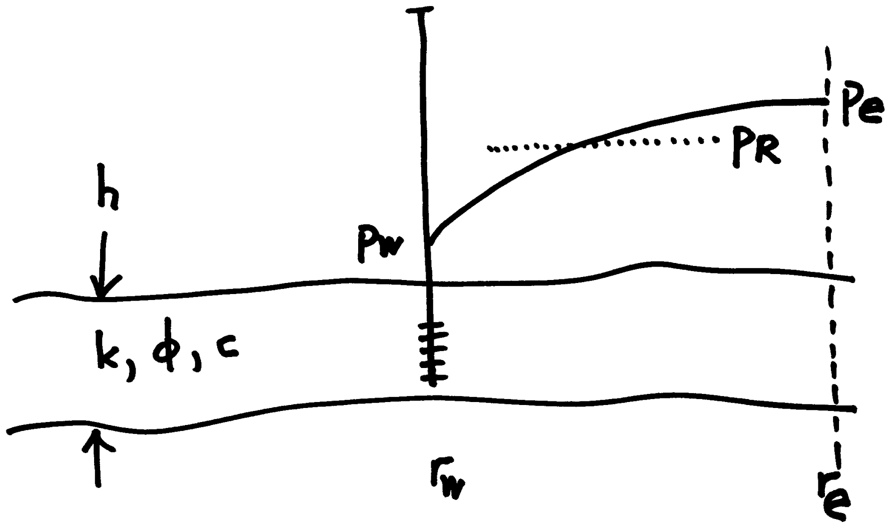
Oil and gas reservoirs

- Reservoirs are almost like a flat pancake with properties k, φ, h and c.
- Wells have property s.
- Well testing (pressure transient testing) gives the groups kh (permeability thickness), representing the flow capacity, while φch (porosity-compressibility thickness) gives the storage capacity (of oil and/or gas)
- GOC is the gas-oil contact and WOC is the water-oil contact.



Pressure profile in reservoir

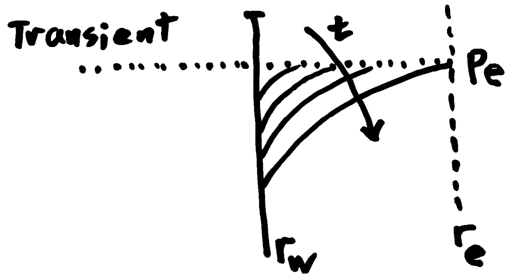
Pressure profile from r_w til r_e, from well radius to outer boundary (e for exterior). Drainage area (and drainage volume) extends from well to outer boundary.

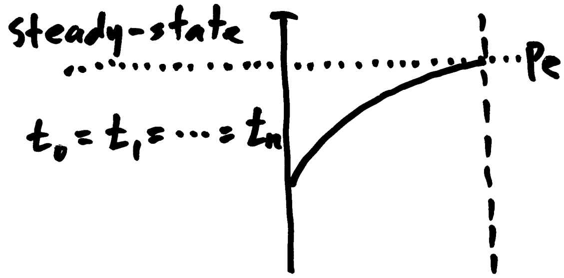


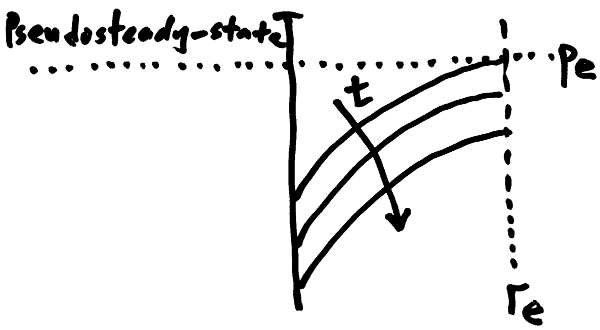
k = permeability [m^2]
φ = porosity [-]
c = compressibility [Pa^{-1}]
h = reservoir thickness [m]
r_w = well radius [m]
r_e = reservoir radius (radial system, e = exterior) [m]
$p_w = p_{wf}$ well pressure (wf = well flowing) [Pa]
p_e = pressure at outer boundary (exterior) [Pa]
p_R = reservoir pressure [Pa]

Pressure states in reservoirs

- Transient state (TS)
- Steady-state (SS)
- Pseudosteady-state (PSS)







Drainage area

PSS occurs when pressure profiles meet (at the drainage boundary). It is the most common state in production of fluids from subsurface reservoirs. TS occurs when wells are started-up or shut-down. The TS is used to test the flow properties of wells, both the flow capacity and storage capacity of the formation.

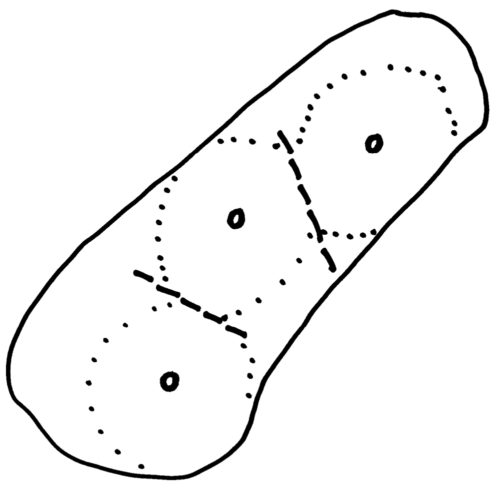
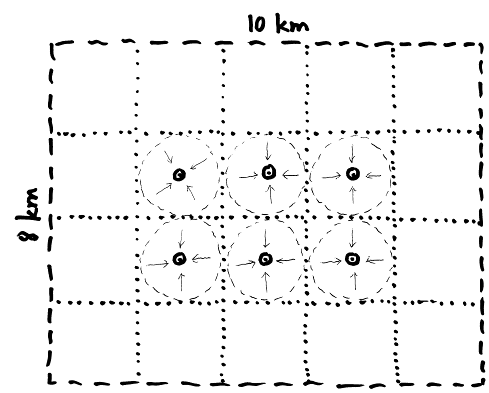


Figure - Areal view of a reservoir with three production wells, illustrating the drainage area and drainage boundary.



Darcy's law

Assuming steady-state flow through porous media, the flowrate (expressed as fluid velocity) is proportional to permeability and pressure gradient. The more viscous the fluid, the lower the flowrate. Viscosity oil 0.5 [mPa.s=cp], viscosity natural gas 0.02 [mPa.s], differ by factor of 25.

$$u = -\frac{k}{\mu}\frac{dp}{dr}$$

u = Darcy velocity = filtration velocity (based on the whole area; that is, not only the pore spaces). It is an analogy to superficial velocity in two-phase flow in pipes.

q = uA

A = 2πrh

Integrate from r and p to r_w and p_{wf}, thus minus cancelled out.

$$p = p_{wf} + \frac{\mu q B}{2\pi k h}\ln\left(\frac{r}{r_w}\right)$$

$$q = q_{s.c.}B$$

$$B = \frac{V}{V_{s.c.}}$$

- Reservoirs have properties k, φ, h, c
- Wells have property s
- Well testing gives kh (permeability thickness), the flow capacity
- Well testing gives φch (porosity-compressibility thickness), the storage capacity

Volumetric flow q stands for rate at local conditions (*in situ*). Volumetric flow $q_{s.c.}$ stands for rate at standard conditions. In the U.S. these are 1 atmospheric pressure and 60 F, while in Norway these are 1 atmospheric pressure and 15 C.

The factor B is the formation volume factor. It expresses how the volume of fluid changes from reservoir to surface separator. It is quite important in oil production because it takes into account the volume reduction of oil as dissolved gas evolves.

The concept of relative permeability is not discussed here. Oil hampers the flow of gas, and gas hampers the flow of oil.

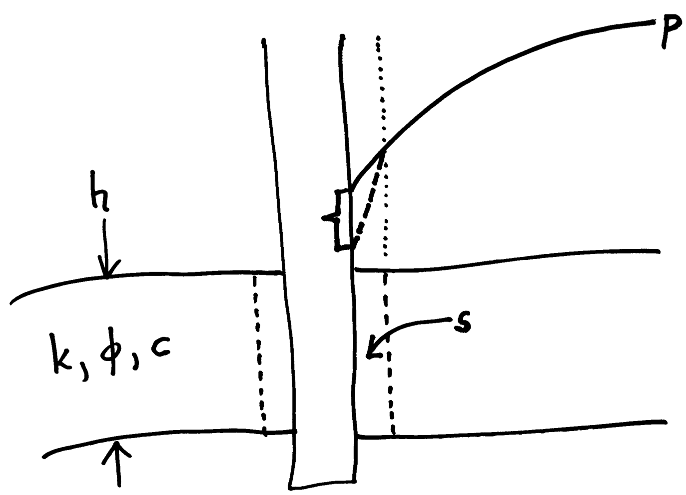
Skin factor

- Increases or decreases the pressure drop near wellbore
- Expresses pressure drop deviating from an ideal well behaviour
- Constant for a particular well
- Damage, near-wellbore, s >0
- Stimulated, near-wellbore, s<0
- Geometric skin, s>0
- Fracture skin, s<0
- Deposits, s>0

The skin factor can arise because of several types of skin (damage, fracture, geometry etc.). In such cases the skin factor is constant. There can also be a rate dependent skin, for example due to high-velocity flow effects. Rate dependent skin is not discussed her.

Constant skin can be expressed in terms of apparent well radius as

$$r_{wa} = r_w e^{-s}$$



Superposition

- The total pressure drop at any point in a reservoir is the sum of the pressure drops at that point caused by the flow in each of the wells in the reservoir.
- Applies in all pressure states, TS, SS and PSS.
- The diffusivity equation describes the pressure distribution in a reservoir with time and distance. The solution to the diffusivity equation is linear; therefore, the pressure values can be added.
- The principle of superposition can also be used to take into account the presence of faults, by using image wells.

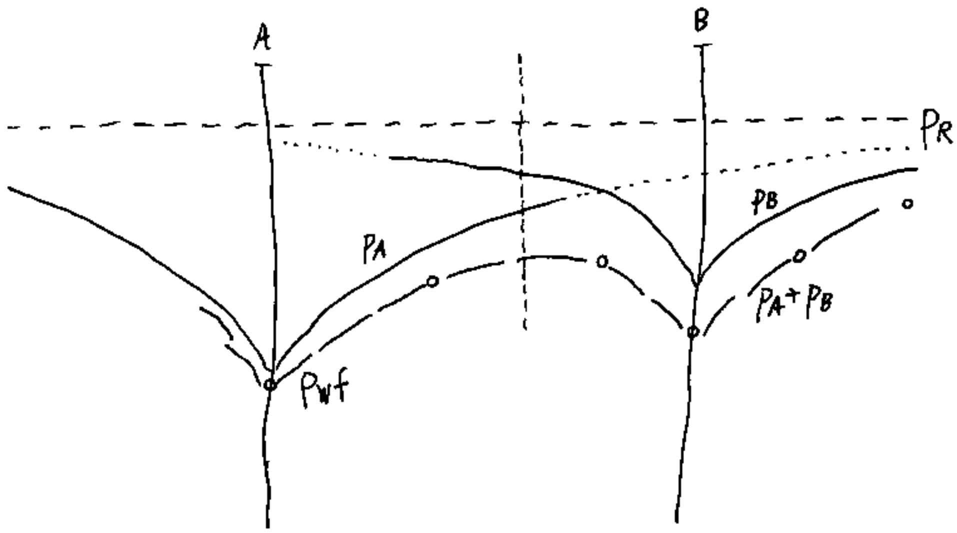


Figure – Pressure profiles for well A and well B alone (individually) are shown as p_A and p_B. In superposition the individual drawdowns are added, here shown as p_A+p_B. The drainage boundary between the wells is shown as a vertical line.

Well testing

- Pressure transient analysis, constant q, variable p
- Decline curve analysis, constant p, variable q
- Step rate testing, p and q at different near-steady values

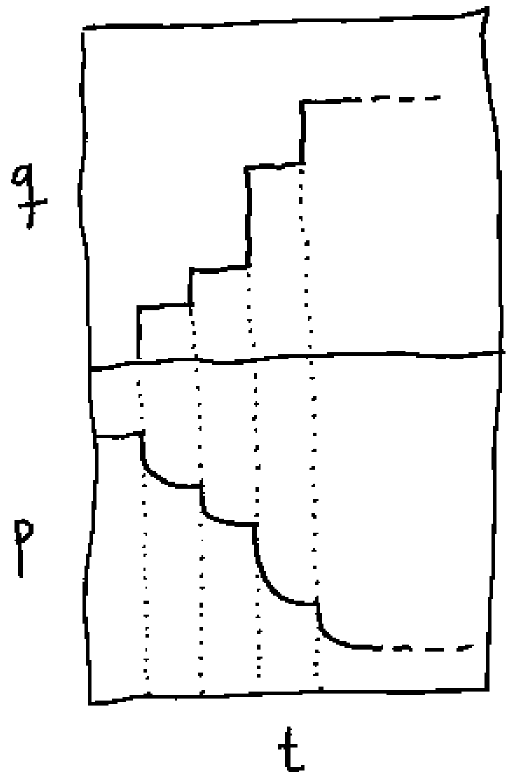


Figure – Step rate testing, each interval step change run long enough for stable q and p.

Dimensionless numbers

Used in pressure transient analysis.

$$p_D = \frac{2\pi kh}{qB\mu}\Delta p$$

$$t_D = \frac{kt}{\phi\mu c_t r_w^2} = \frac{kh}{\phi c_t h}\frac{t}{\mu r_w^2}$$

$$r_D = \frac{r}{r_w}$$

Liquid reservoir performance

- Pressure with time (cumulative production)
- Usual state PSS (when no injection)
- p_R decreases with time, or with N_p, cumulative production
- p_i is initial reservoir pressure (at some reference depth)

$$N_p B = V c_t (p_i - p_R)$$

V is the fluid filled pore volume and c_t the total (formation and fluid) compressibility.

$$c_t = c_f + S_o c_o + S_g c_g + S_w c_w$$

$$p_R = p_i - \frac{qB}{A\phi c_t h} t$$

Isothermal compressibility of fluid is defined by

$$c = \frac{-1}{V}\left(\frac{dV}{dp}\right)_T$$

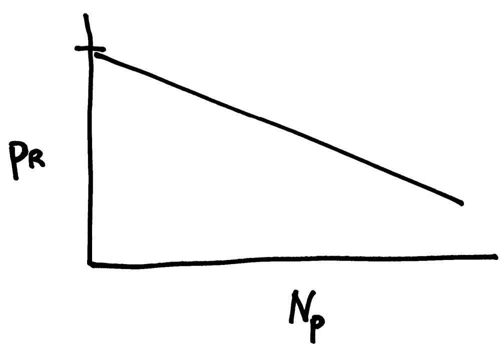


Figure - Reservoir pressure decreases with increased liquid production. The recoverable liquid is less than the liquid originally in place. Water influx makes reservoir pressure decrease less rapidly.

It is less helpful to plot p_R with time because the fluid withdrawal rate most likely varies with time, e.g. from winter to summer. On a p_R with time t plot, therefore, the line will not be as straight as on a p_R with cumulative production plot.

Above the fluid is produced due to expansion from reservoir pressure to wellbore pressure (confined reservoir volume). The reservoir pressure in oil reservoirs can be maintained/increased by injecting gas above the oil zone and/or water (brine) below the oil zone.

Liquid inflow performance

- PSS rate equation

$$q = \frac{2\pi kh(p_R - p_{wf})}{\mu B\left[\ln\left(\frac{r_e}{r_w}\right) - 3/4 + s\right]}$$

- Productivity index

$$q = PI(p_R - p_{wf})$$

$$PI = \frac{2\pi kh}{\mu B\left[\ln\left(\frac{r_e}{r_w}\right) - 3/4 + s\right]}$$

- PSS pressure equation

$$p_{wf} = p_R - \frac{\mu qB}{2\pi kh}\left[\ln\left(\frac{r_e}{r_w}\right) - 3/4 + s\right]$$

Reservoir pressure p_R above and elsewhere is the volume average reservoir pressure given by

$$p_R = \frac{\int_{r_w}^{r_e} p dV}{\int_{r_w}^{r_e} dV}$$

The constant ¾ arises when the volume average pressure is used in PSS. In SS the constant is ½.

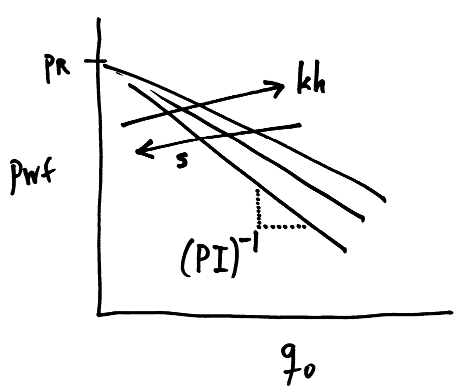


Figure – Increasing kh increases flowrate, increasing s decreases flowrate. Gradient is equal to (-1/PI, minus missing in figure).

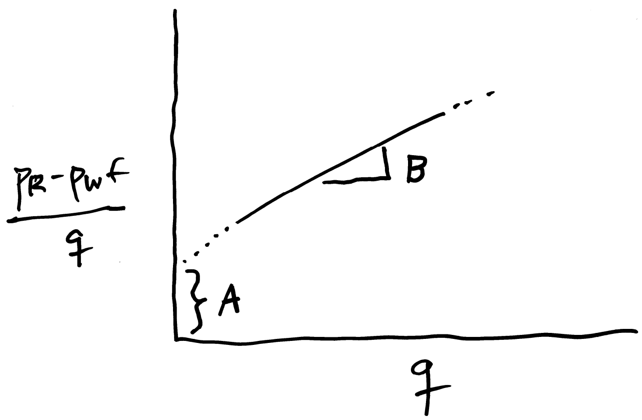
Step rate testing of wells

Liquid systems:

$$p_R - p_{wf} = Aq + Bq^2$$

The first part on the RHS, Aq, expresses the pressure drop due to Darcy flow. The second part on the RHS, Bq^2, expresses the pressure drop to high-velocity flow. Dividing by q gives

$$\frac{p_R - p_{wf}}{q} = A + Bq$$



Low-pressure natural gas:

$$\frac{p_R^2 - p_{wf}^2}{q} = A + Bq$$

The backpressure equation

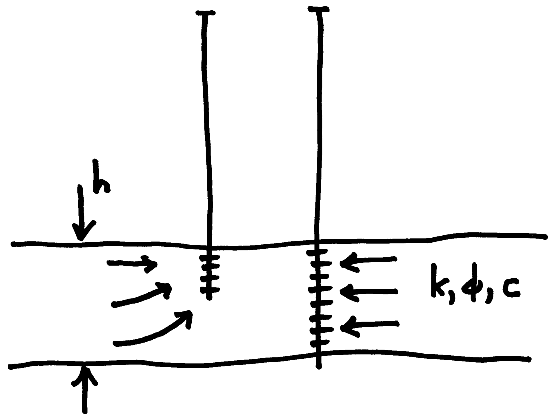
$$q = C\left(p_R^2 - p_{wf}^2\right)^n$$

can also be used for low pressure gas wells. C and n are the backpressure coefficient and exponent, respectively. The value of n is usually in the range 0.5 to 0.7.

Skin and deliverability

- Reservoir performance
- Inflow performance
- Outflow performance

Imagine two nearby wells in a homogeneous reservoir. One of the wells is fully penetrating while the other well is partially penetrating. The production tubing in the wells are identical. Therefore, the reservoir performance and the outflow performance of the two wells are the same, but the inflow performances are different; this due to geometric skin.



Density of gas, standard conditions and z-factor

Real gas law

$$pV = znRT$$

Gas density

$$\frac{n}{V}\left(mol/m^3\right) = \frac{p}{zRT}$$

$$\rho\ \left(kg/m^3\right) = \frac{pM}{zRT}$$

$$M\left(kg/kmol\right)$$

Standard conditions (s.c.)

$$n = n_{s.c.}$$

$$\frac{pV}{zRT} = \frac{p_{s.c.}V_{s.c.}}{z_{s.c.}RT_{s.c.}}$$

$$z_{s.c.} = 1$$

$$V = V_{s.c.}\left(\frac{p_{s.c.}}{p}\right)\left(\frac{T}{T_{s.c.}}\right)z$$

$$q = q_{s.c.}\left(\frac{p_{s.c.}}{p}\right)\left(\frac{T}{T_{s.c.}}\right)z$$

$$B_g \text{ (=FVF gas)} = \frac{V}{V_{s.c.}} = \left(\frac{T}{T_{s.c.}}\right)\left(\frac{p_{s.c.}}{p}\right)z$$

Natural gas inflow performance

Rate equation from Darcy's Law

$$q = \frac{2\pi kh}{\ln\left(\frac{r_e}{r_w}\right)}\left(\frac{T_{s.c}}{T}\right)\left(\frac{1}{p_{s.c}}\right)\int_{p_{wf}}^{p_e}\left(\frac{p}{\mu_g z}\right)dp$$

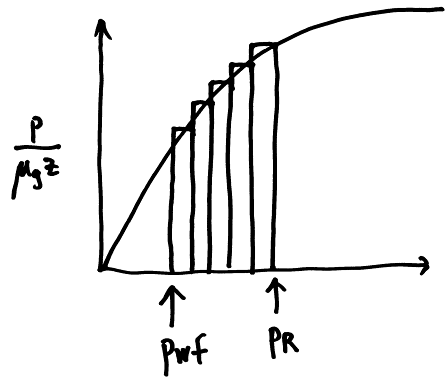
Rate equation from Darcy's Law for PSS and skin. Note that the integration is from p_{wf} (well flowing pressure) as above but here to p_R (volume average reservoir pressure)

$$q = \frac{2\pi kh}{\left[\ln\left(\frac{r_e}{r_w}\right) - 3/4 + s\right]}\left(\frac{T_{s.c.}}{T}\right)\left(\frac{1}{p_{s.c.}}\right)\int_{p_{wf}}^{p_R}\left(\frac{p}{\mu_g z}\right)dp$$

- Solution method, the pressure function (can also be called property function), because gas viscosity and z-factor change with pressure

$$F(p) = \left(\frac{p}{\mu z}\right)$$

Integration numerically between p_R and p_{wf}



- Specific/limiting solutions

  Low pressure, pressure function increases linearly with pressure such that (as in the back-pressure equation).

  $$"\Delta p" = p_R^2 - p_{wf}^2$$

  High pressure, pressure function constant (same expression as for oil)

  $$"\Delta p" = p_R - p_{wf}$$

The pressure function (property function) at low pressure is linear with pressure. Therefore

$$\int_{p_{wf}}^{p_R} \left(\frac{p}{\mu z}\right) dp = \left(\frac{1}{\mu z}\right) \int_{p_{wf}}^{p_R} p dp = \left(\frac{1}{\mu z}\right) \frac{1}{2} \left(p_R^2 - p_{wf}^2\right)$$

Note that the factor 2 cancels out when this expression is used in the rate equation for natural gas wells.

Natural gas reservoir performance

$$pV = zmRT$$

$$n = \frac{W}{M}$$

where W is mass of natural gas. Assume reservoir V constant and reservoir T constant

$$pV = z\frac{W}{M}RT$$

$$\frac{p}{z} \propto W$$

$$\frac{p}{z} = \frac{p_i}{z_i}\left(\frac{W_i - W}{W_i}\right)$$

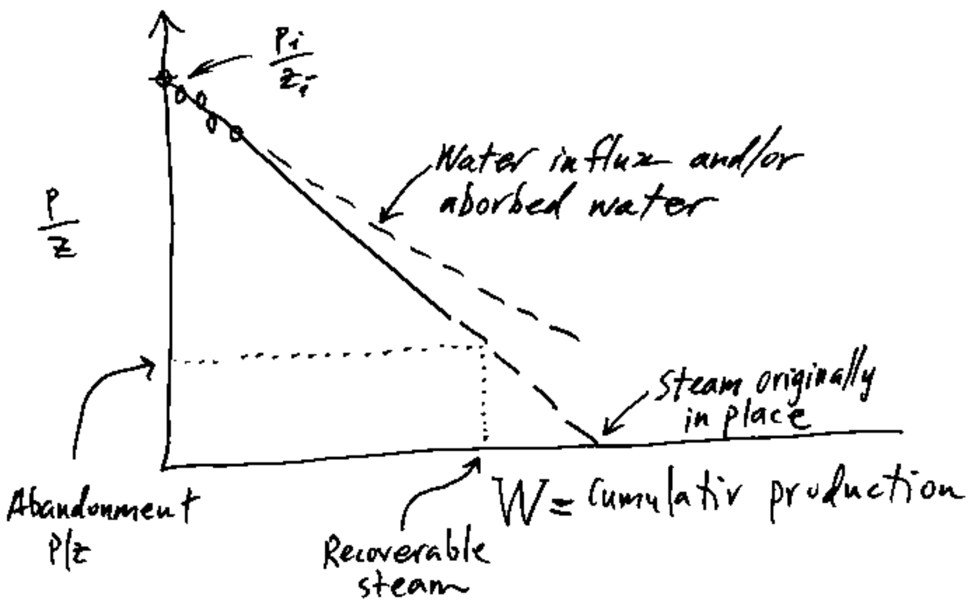


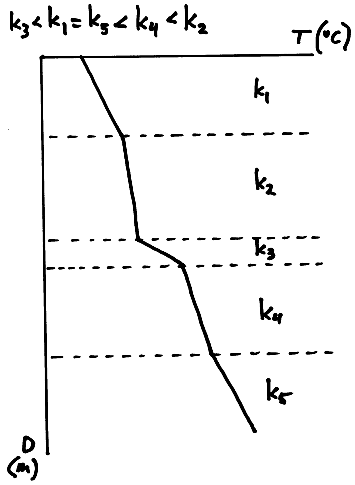
Figure – p/z method for vapour dominated steam reservoirs. Same as for natural gas reservoirs.

Reservoir temperature

- Temperature with depth
- Figure immediately below shows conduction only (no convection)
- Fourier's Law

$$q = -kA\frac{dT}{dx}$$

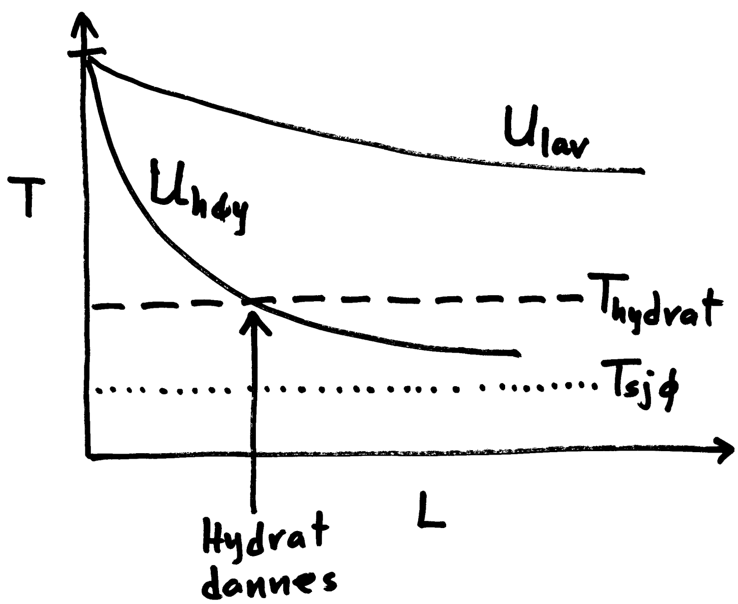
q = heat flow [W = J/s]
A = area [m^2]
k = thermal conductivity [W/m.K]
T = temperature [K]
x = distance [m]



Temperature in pipes and wells

- Inlet T_1, outlet T_2, environment (air, soil, water) T

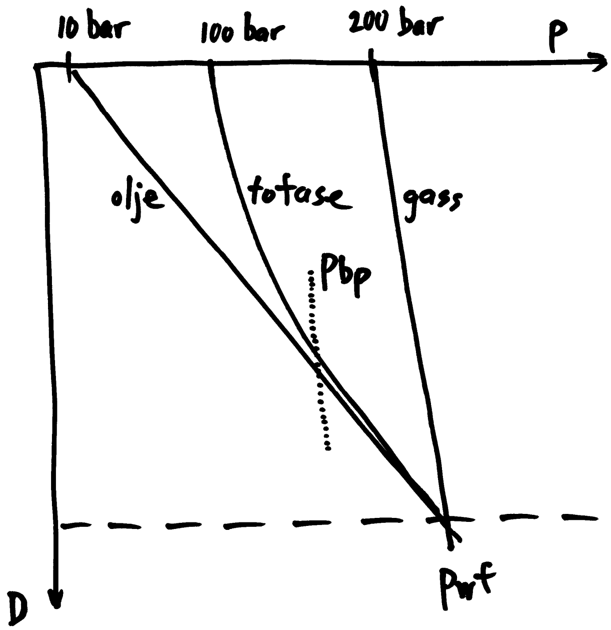
$$T_2 = T + (T_1 - T)\exp\left[\frac{-U\pi d}{mC_p} L\right]$$



- Insulated pipelines, $1 < U < 2$ (W/m^{2}K)
- Non-insulated pipelines, $15 < U < 25$ (W/m^{2}K)

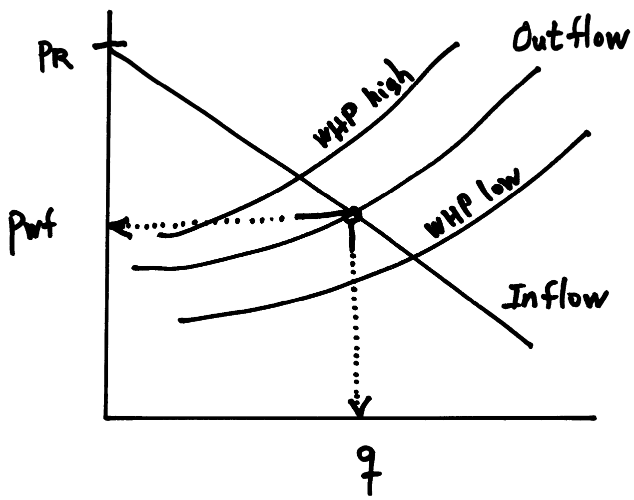
Pressure profiles in flowing wells

- Liquid only (oil or water)
- Vapour only (gas or steam)
- Two-phase oil well with bubble point in production tubing



Outflow Performance

- Outflow performance, also called vertical lift performance
- Pressure drop measured/calculated from wellhead to bottomhole (from p_{th} = tubing head pressure to p_{wf})
- Analytical equations and/or wellbore flow packages can be used for calculations
- Each curve for each wellhead pressure (and, one production tubing design)
- Production rate given by point where inflow and outflow curves meet.



Static pressure in wells

- Liquid only

$p = \rho g L$

- Vapour only (use average values of z and T, from wellhead to bottomhole)

$$p = p_o \exp\left[\frac{gM}{\bar{z}\bar{R}\bar{T}} L\right]$$

To solve, assume average z and average T, then iterate.

Pressure drop in pipes and wells

- Total pressure drop, friction + hydrostatic (gravity) + acceleration
-

$$\Delta p = \Delta p_f + \Delta p_g + \Delta p_a$$

- Pressure drop due to acceleration

$$\Delta p_a = \rho u \Delta u$$

- Hydrostatic pressure drop

$$\Delta p_g = \rho g \Delta h$$

where Δh is the height difference between outlet and inlet; that is, not thickness of reservoir.

- Darcy-Weisbach equation for liquids (incompressible fluid), wall friction

$$\Delta p_f = \frac{f}{2}\frac{L}{d}\rho u^2$$

- Compressible fluids, wall friction
-

$$\frac{d\,A^2\,M}{f\,m^2\,zRT}\left(p_2^2 - p_1^2\right) - \frac{d}{f}\ln\left(\frac{p_2^2}{p_1^2}\right) + L = 0$$

For typical pipelines, the logarithmic part can be ignored.

- Friction factor

Smooth pipes, Blasius' equation when Re < 10^5.

$$f = \frac{0{,}316}{\mathrm{Re}^{0{,}25}}$$

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

Rough pipes, Haaland's equation, for general use.

$$\frac{1}{\sqrt{f}} = -\frac{1{,}8}{n}\log\left[\left(\frac{6{,}9}{\mathrm{Re}}\right)^{n} + \left(\frac{k}{3{,}75d}\right)^{1{,}11n}\right]$$

n = 1 for liquids, n = 3 for gas and steam

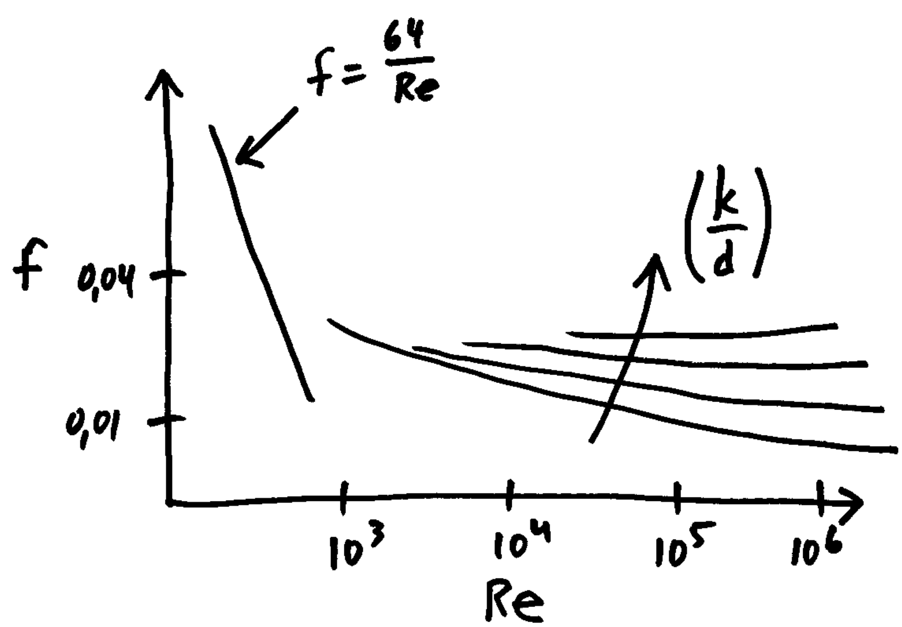


Table below is for Oil Country Tubular Goods.

Material	Average Absolut Roughness (inch)	Average Absolut Roughness (μm)
Internally plastic coated pipeline	0.200×10-3	5.1
Honed bare carbon steel	0.492×10-3	12.5
Electropolished bare 13Cr	1.18×10-3	30.0
Cement lining	1.30×10-3	33.0
Bare carbon steel	1.38×10-3	35.1
Fiberglass lining	1.50×10-3	38.1
Bare 13Cr	2.10×10-3	53.3

Friction factor equations (Haaland and Colebrook-White) give approximately the same friction factor. Experience shows that real pressure drop is less than calculated pressure drop. The reason is not that the friction factor equations are wrong, rather, roughness values recommended (e.g. NORSOK) are too high.

Pumps and pumping

Types of pumps
- Volumetric (piston pump)
- Dynamic (centrifugal pump)
  - Rate depends on diameter
  - Pressure depends on number of stages (wheels)

Pressure and head are used

$$p = \rho g h$$
$$h = \frac{p}{\rho g}$$

Ideal pump power (unit W)

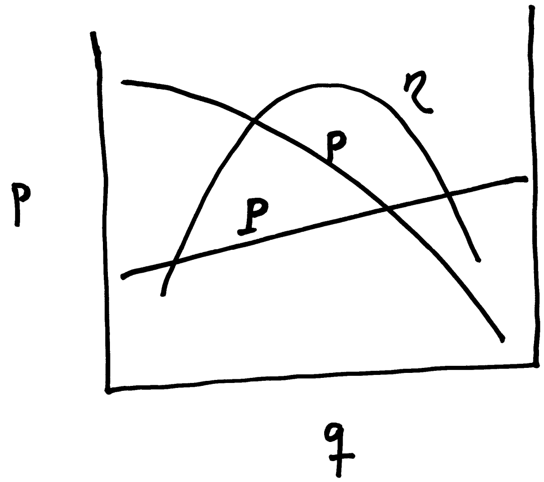
$$P = q\Delta p$$

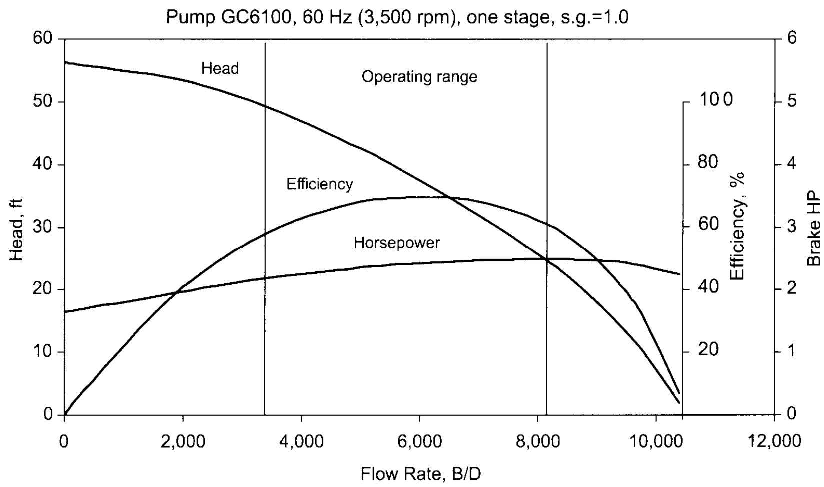
Real pump power

$$P = q\Delta p \frac{1}{\eta}$$

where η is efficiency, typically 0.8 at design conditions.

Characteristic curve (centrifugal pump)

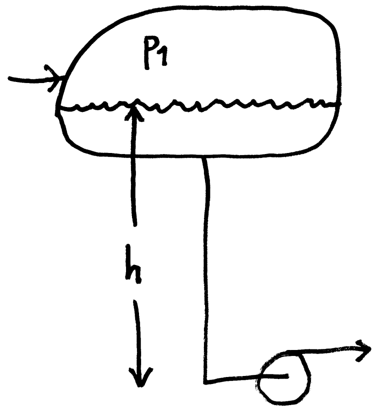




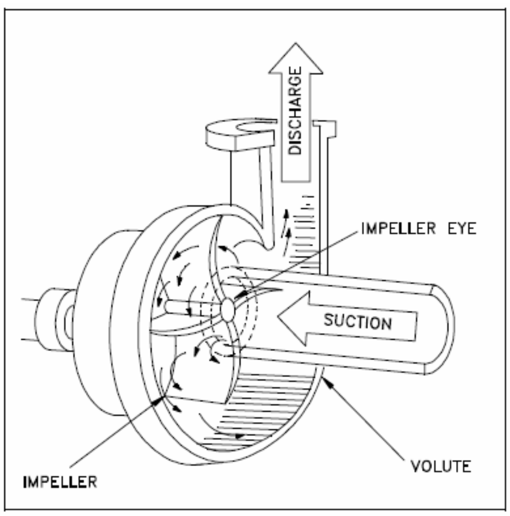
Curves from manufacturer (Pessoa & Rado, SPE Production & Facilities, February 2003)

Cavitation and suction head

- All pumps must have high enough inlet pressure, otherwise the fluid will start to boil
- Cavitation, bubbles created and collapse
- Head to a pump is height minus friction (from open tank)
- NPSH = Net Positive Suction Head, specified by manufacturer
- When p_1 is equal to p_s (saturation pressure) a special case
- Heat of vaporisation of water is higher than for hydrocarbons (pumps pumping oil are more likely to cavitate than pumps pumping water/brine)



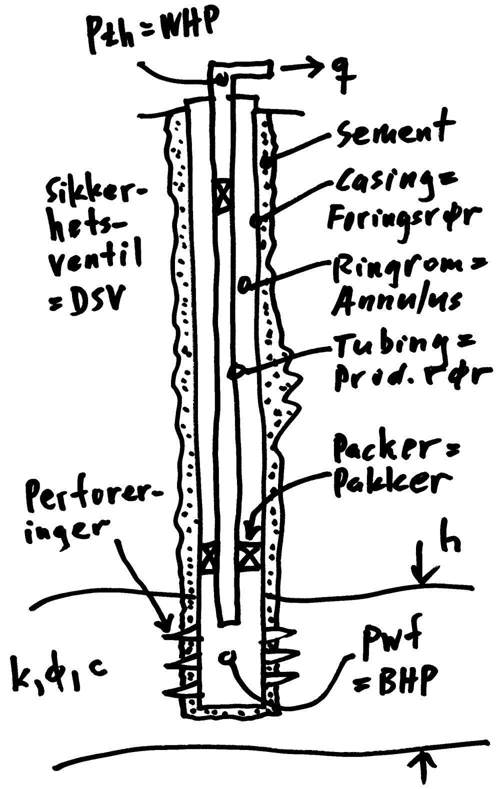
$$NPSH = h + \frac{p_1 - \Delta p_f - p_{sat}}{\rho g}$$



Centrifugal pump (DoE 1993).

Well design

- Casing cemented from wellhead to bottom
- Casing perforated in oil producing formation(s)
- Production tubing inside the casing (annulus between)
- Packer at bottom of tubing to seal between casing and tubing
- Production through perforations and up the tubing.
- Downhole safety valve, typically at 200-400 m depth



Wellhead and manifold

- Two master valves (either fully open or fully closed)
- One valve on top for logging operations (otherwise closed)
- Wing valve on horizontal leg, used to open and closed well)
- Choke valve after the wing valve (controls the well, flow and pressure)
- All wells feed to manifold, and after that to process separator
- Individual wells can be coupled from manifold and to test separator

Artificial lift

With and without ESP (electrical submersible pump)

- Vertical pressure profile, p_t (pressure in tubing)
- Wellhead pressure, p_{th} (= WHP)

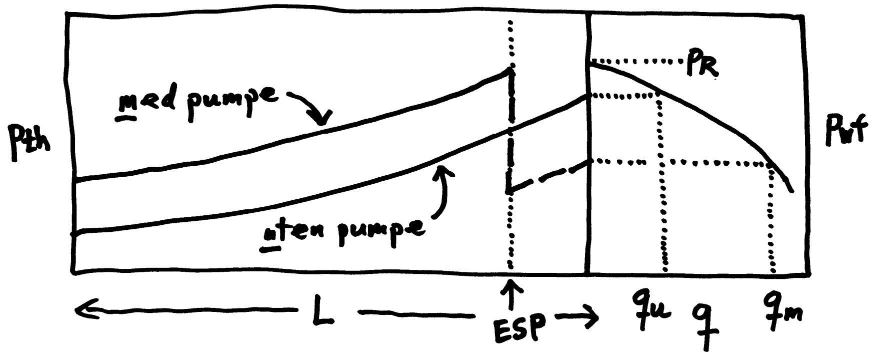
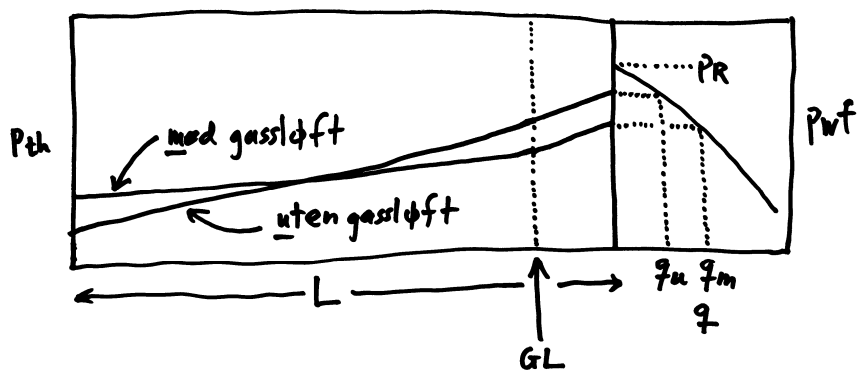
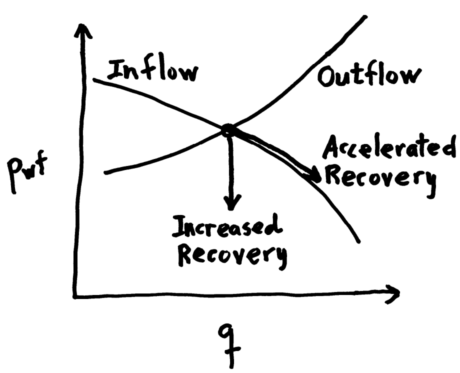


Figure below shows gas lift.





Downhole pumps can increase the flow rate and Accelerate Recovery and/or Increase Recovery.

APPENDIX A: Pressure transient analysis

The transient state of a well can last for less than an hour to more than several days. During this time the well pressure and flow rate change with time. How the pressure and flow rate change with time depends on the nature of the well-reservoir system. If the flow rate is kept constant during the transient state, pressure transient analysis can be performed, also call well test analysis. If the well pressure is kept constant during the transient state, decline curve analysis can be performed. In real testing situations it is difficult to keep the flow rate or well pressure constant. In modern well test analysis the principle of superposition and appropriate computer programme are used to compensate for variations with time in well pressure and flow rate. To illustrate the use of pressure and/or flow rate transients it is convenient to assume one of these parameters as constant. In the following, the flow rate is assumed constant, as in traditional pressure transient analysis.

The transient state can be used to extract information about a well-reservoir system, including:

- Permeability-thickness kh in drainage volume of well
- Porosity-thickness-product φch or storativity of drainage volume
- Condition of well represented by skin factor s (whether well is damaged or stimulated)
- Average reservoir pressure p_R within the drainage volume, at the time of testing
- Reservoir geometry, faults, fractures, gas-liquid interfaces.

Pressure transient analysis as presented here is that used by petroleum engineers in the oil and gas industry. Pressure transients are also used in the groundwater hydrology industry. While the theoretical basis is the same in both industries, the terminology is not identical.

In the petroleum industry Darcy's Law is commonly written

$$u = \frac{-k}{\mu}\frac{dp}{dr}$$

while in groundwater hydrology it is commonly written

$$u = -K\frac{dh}{dr}$$

because the concept of head, h, is used. The symbol K stands for permeability and is related to intrinsic permeability by

$$k = K\frac{\mu}{\rho g}$$

In the present text the intrinsic permeability k (m^2) will be used and called, just, permeability. This permeability is sometimes called hydraulic conductivity. The permeability K (m^2/s) in groundwater hydrology is used in the definition of transmissivity T = Kh.

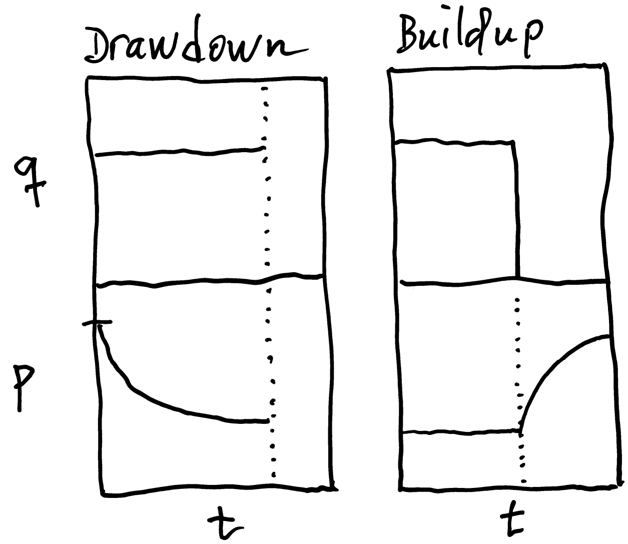
Several types of tests are used in pressure transient analysis. Single well tests include:

- Drawdown test, production well
- Buildup test, production well
- Injectivity test, injection well

- Falloff test, injection well

Multiple well tests include:

- Interference test, active well
- Interference test, observation well



The above figure shows a drawdown test where the flow rate is kept constant and the well pressure decreases with time. The figure shows also a buildup test where the well has been on-stream long enough for the well pressure to stabilize. The well pressure increases when the well is shut-in (closed). Both drawdown and buildup transients can be used to determine the properties of well-reservoir systems.

The partial differential equation that describes the propagation of a pressure wave with time in a reservoir formation subject to fluid production (or injection) is the simplified diffusivity equation

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{k}\frac{\partial p}{\partial t}$$

It is one-dimensional because it includes only radial distance. Pressure changes are assumed small and Darcy's Law applies. The system is slightly compressible and the fluid viscosity is constant. The reservoir engineering parameters on the right-hand side are called hydraulic diffusivity

$$\eta = \frac{k}{\phi \mu c_t}$$

The diffusivity equation is used to describe all kinds of diffusivity processes. The solution of the diffusivity equation involves what is called the exponential integral, E, and can be written as follows

$$p(r,t) - p_i = \frac{-q\mu}{4\pi kh} E\left(\frac{\phi\mu c_t r^2}{4kt}\right)$$

Reservoir pressure at any distance r and at any time t, p(r,t), depends on the flow rate, formation properties, fluid properties and the solution of E. An exact and complete solution of E is not available so approximate solutions are used, appropriate for different time spans. The following solution can be used in pressure transient testing

$$E(x) \cong -\ln(x) - \gamma$$

where γ is Eulers constant 0.5772 (the limiting difference between the harmonic series and the natural logatithm). The E solution is appropriate for $x < 0.01$ representing large values of time or small distances, such as a wellbore.

$$p - p_i = \frac{-q\mu}{4\pi kh}\left[\ln(t) + \ln\left(\frac{4k}{\phi\mu c_t r^2}\right) - \gamma\right]$$

The groups k/μ, kh/μ and φch are in petroleum engineering called mobility, mobility thickness and storativity, respectively.

Using logarithm to base 10 the equation can be written as

$$\boxed{p - p_i = -m\left[\log t + \log\left(\frac{4k}{\phi\mu c r^2}\right) - 0.2507\right]}$$

where

$$\boxed{m = \frac{2.303 q\mu}{4\pi kh}}$$

If the pressure difference p-p_i is plotted against log(t) the slope of the line will be m (determined from plot). Assuming the flow rate q is constant, the mobility thickness can be determined from

$$\boxed{\frac{kh}{\mu} = \frac{2.303 q}{4\pi m}}$$

At some time t, the solution of the pressure transient equation can be written

$$\frac{p - p_i}{m} = -\log\left[\left(\frac{4kh}{\mu}\right)\left(\frac{1}{\phi ch}\right)\left(\frac{t}{r^2}\right)\right] + 0.2507$$

It follows that the storativity can be determined from

$$\boxed{\phi c h = 2.25\left(\frac{kh}{\mu}\right)\left(\frac{t}{r^2}\right)10^{\left(\frac{p-p_i}{m}\right)}}$$

The above solution of the diffusivity equation has different names in different fields of study. In petroleum engineering it is called line-source solution. In groundwater hydrology it is called Theis solution. In other fields of study the solution is sometimes called Kelvin's point source.

The above equations are presented using SI units. Oil field units are commonly used in the oil and gas industries, such that the constants have different values. Other units can also be used in groundwater hydrology.

In the above equations the radius r is usually the well radius where the pressure is measured. In interference testing, where the pressure is measured at some distance (distance between active well and observation well) the radius value used is the same distance. The compressibility above is the total compressibility c_t. The volumetric flow rate q is the local flow rate (down hole). If flow rate at standard conditions is used, as common, the formation volume factor needs to be applied.

The diffusivity equation is a linear partial differential equation, which means that the pressure values (effects) are additive. Therefore, the principle of superposition can be used. For a well that has produced for time t and then shut-in for time Δt. For time t+Δt the exponential integral solution can be written

$$p(r,t) - p_i = \frac{-q\mu}{4\pi kh} E\left(\frac{\phi\mu c_t r^2}{4kt}\right) - \frac{-q\mu}{4\pi kh} E\left(\frac{\phi\mu c_t r^2}{4k\Delta t}\right)$$

Following the same derivation as above gives the result

$$p - p_i = \frac{2.303q\mu}{4\pi kh}\log\left(\frac{t+\Delta t}{\Delta t}\right)$$

A plot of pressure against the logarithm of the time ratio is called a Horner-plot. When extended/extrapolated to 1, the initial reservoir pressure (average reservoir pressure) can be estimated. Symbol p^* is commonly used for this extrapolated pressure.

The line-source solution can be written using dimensionless numbers, resulting in

$$p_D = \frac{1}{2}\left[\ln t_D + 0.80907\right]$$

The dimensionless equation is used in type-curve matching. It can also be used to illustrate how the skin factor, s, can be included. The skin factor is constant and is dimensionless such that

$$p_D = \frac{1}{2}\left[\ln t_D + 0.80907\right] + s$$

Another way to express the effect of skin is to write the resulting additional pressure drop as

$$\Delta p_s = \frac{q\mu}{2\pi kh} s$$

Because skin represent additional pressure drop it can be added to the line-source solution equation

$$p - p_i = -\frac{2.303 q\mu}{4\pi kh}\left[\log t + \log\left(\frac{4k}{\phi\mu c r^2}\right) - 0.2507 + 0.8686 s\right]$$

In principle, any point on a semilog (log-linear) line can be used to determine skin factor s. It is customary in the petroleum industry to calculate when the logarithm of 1 is zero. Using SI unit, the above equation can be solved for the skin value, s, using all other measured values at 1 second. But at 1 second the well has not reached the time span where the line-source solution applies. The line-source solution results in a straight line on a semilog plot (linear pressure against logarithm of time). Therefore, the straight line must be extrapolated to 1 second. Rearranging the above equation to calculate skin gives

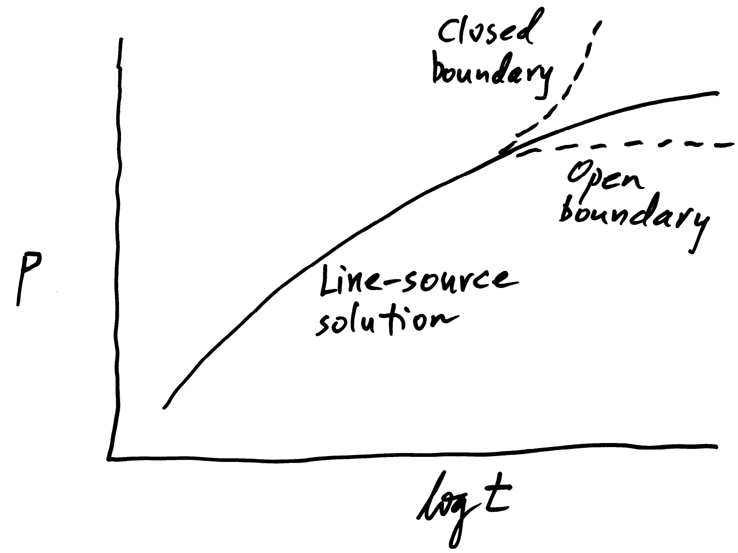
$$\boxed{s = 1.151\left[\frac{p_i - p_{1\sec}}{m} - \log\left(\frac{4k}{\phi\mu c r^2}\right) + 0.2507\right]}$$

When using oil field units, the unit of time is hour, which may result in more realistic values of the measured parameters.

$$s = 1.151\left[\frac{p_i - p_{1hr}}{m} - \log\left(\frac{k}{\phi\mu c r^2}\right) + 3.2274\right]$$

Traditional oil field units are the following:

k	= permeability (md)
h	= thickness (feet)
p_i	= initial reservoir pressure (psi)
p_{wf}	= well flowing pressure (psi)
q	= production rate (STB/d)
B	= formation volume factor (res vol/std vol)
μ	= viscosity (cp)
t	= time (hour)
φ	= porosity (pore vol/bulk vol)
c	= compressibility (1/psi)
r	= radius (ft)



The line-source solution is infinite acting. With time the pressure change will reach the outer boundary of a drainage volume. If the outer boundary is open, the pressure will flatten out (not usual). If the outer boundary is closed, the pressure will change more rapidly (usual, for example PSS). Pressure measurements at the start will also include the effect of wellbore fluid, called wellbore storage; it masks the pressure response of the reservoir formation. Wellbore storage is characterized with unit slope on a log-log plot. The slope m must be taken after the wellbore storage effect has disappeared from the data.

Wellbore storage constant C, drainage volume shape factor C_A are not covered in the present lecture notes, neither is skin due to high-velocity flow, Dq.

Conversion Factors

Traditionally units	SI-equivalents	Useful info.
LENGTH	**M**	
mile (mi)	1609.344 m #	grunnenhet
yard (yd)	0.9144 m #	
foot (ft)	0.3048 m #	3 ft = 1 yd
inch (in)	0.0254 m #	12 in = 1 ft
VOLUME	**m3**	
US-gallon (gal)	0.00378541 m^3	
UK-gallon (gal)	0.00454609 m^3	
API barrel (bbl)	0.158987 m^3	1 bbl = 42 US gal
kubikkfot (cf)	0.0283167 m^3	1 bbl ~ 5.62 cf
MASS	**kg**	
pound (lbm)	0.45359 kg	
US-ton (ton)	907.185 kg	grunnenhet
UK-ton (ton, tonne)	1016.05 kg	
TEMPERATURE	**Kelvin: K**	
Rankin (R)	5/9 K #	grunnenhet
Celciusgrader (C)	K = C + 273	
Fahrenheit (F)	C = (F-32) · 5/9	R = F + 460
ENERGY, WORK	**Joule: J**	
kalori (cal)	4.184 J #	
erg	10E-7 J	J = Nm
British Termal Unit (BTU)	1055.06 J	
kilowattime (kwh)	3600 J	
POWER	**Watt: W**	
hestekraft (elektrisk) (hk, hp)	746 W #	W = J/s
hestekraft (hydraulisk)	746.043 W	
FORCE	**Newton: N**	
dyn (dyn)	10E-5 N #	N = kg m/s^2
kilopond, eller kilogramkraft (kp/kgf)	9.80665 N	dyn = g cm/s^2
poundforce (lbf)	4.44822 N	
PRESSURE	**Pascal: Pa**	Pa = N/m^2
bar (bar)	105 Pa	1 bar ~ 14.5 psi
pound per square inch (psi)	6894.76 Pa	1 atm ~ 14.7 psi
atmosfære (atm)	1.01325 bar #	1 atm ~ 1.01 bar
mm kvikksølv (torr)	133.322 Pa	
VISCOSITY	**Pa.s**	Pa.s = kg/s.m
poise (p)	10E-1 Pas #	poise =
centipoise (cp)	10E-3 Pas #	dyn/cm^2 s
$lbf/(ft^2/s)$	4.78803 Pas	
DENSITY	**kg/m^3**	
API-gravity (API)	kg/m^3 = (141.5)(1000)/(131.5 + API)	
g/cm^3	1000 kg/m^3 #	
lbm/US-gal	119.826 kg/m^3	
lbm/UK-gal	99.7763 kg/m^3	
lbm/ft^3	16.0185 kg/m^3	
VOLUME FLOW	**m^3/s**	
liter pr. sek. (l/s)	10E-3 m^3/s	
fat pr. dag (bbl/d)	1.8401E-6 m^3/s	
kubikkfot pr. dag (cf/d)	3.2774E-7 m^3/s	
US-gallon pr. minutt (gal/min)	6.30903E-5 m^3/s	

#: Exact values.