DARCY-WEISBACH EQUATION

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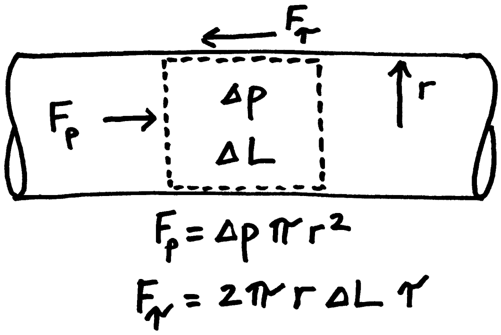
Consider an element of length *dL* subjected to a pressure drop *dp* in a pipeline. The pressure drop acts on the flow area πr^2 while the wall shear stress τ_w acts on the wall area $2\pi r dL$ (circumference times length). The wall shear stress has the unit [N/m^2].



A force balance can be written as follows

$$dp \pi r^2 = 2\pi r dL \, \tau_w$$

such that the wall shear stress can be expressed by

$$\tau_w = \frac{r}{2}\frac{dp}{dL}$$

According to tradition, the wall shear stress in pipes is related to kinetic energy by the empirical equation

$$\tau_w = \frac{1}{8} f \rho u^2$$

where f is a friction factor. Shear stress and kinetic energy per volume have the same unit of [N/m^2=J/m^3] and we can write

$$\frac{r}{2}\frac{dp}{dL} = \frac{1}{8} f \rho u^2$$

Solving for pressure drop gives the well know Darcy-Weisbach equation for pressure drop in pipes

$$\Delta p_f = \frac{f}{2}\frac{L}{d}\rho u^2$$

The pressure drop is given the subscript *f* to indicated frictional pressure drop and *L* is the pipe length. The equation can be used for both laminar flow and turbulent flow in pipes.

The Darcy-Weisbach equation is used for non-compressible flow, such as oil and water. However, it can be used to estimate the pressure drop in natural gas pipe flow (compressible flow), provided an average friction factor and an average density are used.

$$\Delta p_f = \frac{\overline{f}}{2}\frac{L}{d}\overline{\rho}u^2$$