PARTICLE DIFFUSIVITY IN TURBULENT PIPE FLOW

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The distance a particle will travel through a stationary fluid, when subjected to some initial velocity u_o, is the stopping distance:

$$s = \tau u_o \qquad \text{.... (1)}$$

where τ is the particle relaxation time (Fuchs 1964). In studies of particle deposition in turbulent flow three regimes may be identified where the deposition process is controlled by different mechanisms. These regimes are given in terms of the dimensionless relaxation time:

$$\tau^+ = \frac{\tau (u^*)^2}{\nu} \qquad \text{.... (2)}$$

where u^* is the friction velocity and ν the kinematic viscosity. The values of τ^+ for different regimes are given in Table 1.

For particle deposition in the inertia regime Liu and Ilori (1974) have derived a semi-empirical expression for the particle eddy diffusivity:

$$\varepsilon_p = \varepsilon + \tau . (u')^2 \qquad \text{.... (3)}$$

where ε is the eddy diffusivity and u' the root-mean-square fluctuating velocity of the fluid in the direction of the wall. This expression was used in the diffusion equation and solved with the boundary conditions specified by the stopping distance model of Friedlander and Johnstone (1957).

In the general mixing length theory the eddy diffusivity of a lump of fluid is given by:

$$\varepsilon = l u' \qquad \text{.... (4)}$$

where l is the general mixing length (Kay 1968). The particle eddy diffusivity expression derived by Liu and Ilori (1974) can therefore be obtained by equating the stopping distance and the general mixing length:

$$l u' = s u' = \tau (u')^2$$

and adding this term to the eddy diffusivity of the fluid.

The basic parameter that determines the behaviour of particles in turbulent flow is the inertia parameter (Lilly 1973, Williams and Hedley 1975):

$$\psi = \frac{\tau}{\lambda} \qquad \text{.... (5)}$$

where λ is the Lagrangian integral time scale of the fluid turbulence (Hinze 1975). When the inertia parameter $\psi << 1$ a particle will completely follow the turbulent fluctuations of the fluid. If $\psi \sim 1$ a particle will probably fail to follow at least, the higher frequency fluctuations, and when $\psi > 10$ a particle is virtually unaffected by the turbulent fluctuations (Lilly 1973). For homogeneous and isotropic turbulent flow Hinze (1975) has developed the expression:

$$\lambda = \frac{\varepsilon}{(u')^2} \qquad \text{.... (6)}$$

Although turbulent pipe flow is far from homogeneous and isotropic especially close to the wall, the above expression should approximate the conditions in the boundary layer.

A substitution of Equation (6) into Equation (5) results in the following approximation for the inertia parameter:

$$\psi = \frac{\tau\,(u')^2}{\varepsilon} \qquad \text{.... (7)}$$

A further substitution of this expression into Equation (3) shows that the particle eddy diffusivity derived by Liu and Ilori (1974) is dependent only on the fluid eddy diffusivity and the particle inertia parameter:

$$\varepsilon_p = \varepsilon(1 + \psi) \qquad \text{.... (8)}$$

In the turbulent core $\psi << 1$ such that $\varepsilon_p \sim \varepsilon$ but closer to the wall ψ becomes greater and $\varepsilon_p > \varepsilon$. It has been shown by Goldschmit et al (1971) that in turbulent jets $\varepsilon_p > \varepsilon$ when particles fail to follow the turbulent fluctuations (Hinze 1971).

By using the values for the dimensionless relaxation time τ^+ in the inertia regime (0.1 - 10) the inertia parameter ψ can be evaluated at any distance away from the wall. Expressions for ε and u' are readily available (Liu and Ilori 1974). It is found that ψ becomes unity when the dimensionless distance $y^+ = yu^*/\nu$ is in the range 1-20, which is inside the boundary layer. In inertia deposition a particle will probably fail to follow the fluid motions somewhere in the slow moving fluid at the wall.

It has been shown that there are three deposition regimes. In the inertia regime the particle eddy diffusivity is expressed by the fluid eddy diffusivity and the particle inertia parameter. The assumption of the stopping distance model, that a particle moves toward the wall as through a stationary fluid after failing to follow the turbulent fluctuations, appears to be borne out by analysis of fluid-particle interactions.

NOTATION

l : General mixing length
s : Stopping distance
u^* : Friction velocity
u_o : Initial velocity
u' : Root-mean-square fluctuating velocity
y : Distance away from wall
y^+ : Dimensionless y
λ : Lagrangian integral time scale
ε : Fluid eddy diffusivity
ε_p : Particle eddy diffusivity
ν : Kinematic viscosity
τ : Particle relaxation time
τ^+ : Dimensionless τ
ψ : Inertia parameter

TABLE 1

Values of τ^+ for Different Regimes

τ^+	Regime	Reference
< 0.1	Diffusion	Beal, S K, (1970) Davies, C N, (1966) Wells, A C, and Chamberlain, A C, (1967)
0.1 - 10	Inertia	Beal, S K, (1970) Cleaver, J W, Yates, B, (1975) Cleaver, J W, Yates, B, (1976) Davies, C N, (1966) Davies, C N, (1966) Friedlander, S K, Johnstone, H F, (1957) Kitamoto, A, Takashima, Y, (1974) Kitamoto, A, Takashima, Y, (1975) Liu, B Y H, Ilori, T A, (1974) Liu, B Y H, Agarwal, J K (1974) Montgomery, T L, Corn, M, (1970) Owen, P R, (1960) Sehmel, G A, (1970) Sehmel, G A, (1971) Wells, A C, Chamberlain, A C, (1967)
> 10	Impaction	Gardner, G C, (1974) Gardner, G C, (1975) Hutchinson, P, Hewitt, G F, Duckler, A E (1971) Hutchinson, P, Brown, D J, (1974) Reeks, M W, (1973)

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