3rd Lerkendal Petroleum Engineering Workshop
Trondheim, January 20-21, 1993

MODEL FOR SOUND SPEED IN MULTIPHASE MIXTURES

L. Dong and J.-S. Gudmundsson
Division of Petroleum Engineering and Applied Geophysics
Norwegian Institute of Technology
7034 Trondheim

ABSTRACT

A model has been proposed for sound speed in multiphase mixtures. The sonic velocity in a multiphase mixture is directly related to the properties of its constituent fluids. Model calculations show that for gas-liquid systems the sonic velocity is dramatically lowered by the presence of a small amount of gas and is strongly pressure-dependent. However, this is not the case for liquid-liquid systems. The calculated sonic velocity has been compared with measured data in two-component systems. It indicates that the process of propagation of sonic waves in gas-liquid mixtures is generally isentropic, at least in two-component systems.

INTRODUCTION

The phenomenon of propagation of sonic waves in multiphase mixtures is of practical important in many areas within petroleum engineering. The sonic velocity of a multiphase mixture is much lower than the sonic velocity of its pure constituents. In numerous geologic situations the sonic velocity of a multiphase system may be of interest: in the research for magma reservoirs, in seismic exploration of geothermal areas, prediction of pressure wave velocity decreases prior to earthquakes, and in inversion of crustal and upper mantle seismic records. Fluid flow characteristics during eruptions of volcanoes and geysers are strongly dependent on the sonic velocity of erupting multiphase fluids (Kieffer, 1977). The low value of the sound speed in a multiphase mixture implies that a relatively low flow velocity is associated with choking. The phenomenon may be used for flow-metering purposes in pipelines and wellbores; for example, estimating void fraction and flow velocity of gas/liquid mixtures (Gudmundsson and Dong, 1992).

The sound speed of a fluid is a thermodynamic property defined as

$$c^2=\frac{\partial p}{\partial \rho} \qquad (1)$$

The evaluation of the partial derivative requires knowledge of the thermodynamic process undergone by the fluid as the sonic wave passes. Sir Isaac Newton made a famous error in 1686 by deriving a formula for sound speed which was equivalent to assuming an isothermal process, as discussed by White (1986). The calculated sound speed was about 20 per cent lower than the measured value for air. He rationalized the discrepancy as being due to dust particles in the air. The error is certainly understandable when we reflect that it was made 180 years before the proper basis was laid for the second law of thermodynamics. It is now realized that the correct process must be isentropic because there are no temperature gradients except inside the wave itself. The correct expression for the sound speed is

$$c^2 = \left.\frac{\partial p}{\partial \rho}\right|_S = \gamma \left.\frac{\partial p}{\partial \rho}\right|_T \tag{2}$$

for any single phase fluid, gas or liquid. The subscripts S and T stand for isentropic and isothermal processes, and γ the ratio of specific heats.

Many analytical models for sonic velocity in gas-liquid mixtures (Karplus, 1958; McWilliam and Duggins, 1970; Henry *et al.*, 1971; and Kieffer, 1977) are based on Eq. (1). They related the density of a mixture to the densities of the gases and liquids. To make the differentiation in Eq. (1) possible the ideal gas law was employed for the gas and an adiabatic state equation for the liquid. The derived expression for the density of the mixture was then differentiated with respect of pressure. It is known, however that properties of natural gases deviate from those of ideal gases, especially when pressure is high. Generally fluids in petroleum engineering are under high pressure and temperature. It is therefore expected that considerable error may occur if this type of models is applied to such circumstances.

In the study presented in this paper, an expression for sonic velocity in multiphase mixtures has been developed. Instead of differentiating the density of a gas/liquid mixture, it relates the sonic velocity of the mixture directly to the properties of the gases and liquids. The calculated sonic velocity has been compared with measured data.

THEORY AND MODEL

Using isentropic compressibility K^S or isothermal compressibility K^T, Eq. (2) can be rewritten as

$$c^2 = \frac{1}{\rho K^S} \tag{3}$$

or

$$c^2=\frac{\gamma}{\rho K^T}=\frac{C_p}{\rho K^T\,C_v} \tag{4}$$

where the superscripts T and S refer to an isothermal and isentropic process, C_p isobaric specific heat, and C_v isochoric specific heat.

The sonic velocity of a fluid or mixture can be related directly to its density, isothermal compressibility and specific heats through Eq. (4). For a multiphase mixture these properties are functions of those of its constituent fluids. Therefore, its sonic velocity can be directly related to those of its pure constituents. Knowing those properties of its constituents the sonic velocity of the mixture can be calculated arithmetically instead of differentially. The following shows how to relate the relevant properties of a mixture to those of its constituent fluids.

Density of a gas/liquid mixture is given by

$$\rho_M=\alpha\rho_G+(1-\alpha)\rho_L \tag{5}$$

where α is the gas/liquid void fraction.

If the liquid phase contains both water and oil, the density of the water/oil mixture can be similarly obtained by

$$\rho_L=\beta\rho_W+(1-\beta)\rho_O \tag{6}$$

where the subscripts W and O stand for water and oil, and β the water/oil volumetric fraction.

Substitution of Eq. (6) into (5) gives the density of a gas/oil/water mixture as

$$\rho_M=\alpha\rho_G+(1-\alpha)[\beta\rho_W+(1-\beta)\rho_O] \tag{7}$$

Isothermal compressibility of gas/oil/water mixtures can be expressed as (Craft *et al.*, 1991):

$$K_M^T=S_GK_G^T+S_OK_O^T+S_WK_W^T \tag{8}$$

where S is the volumetric fraction of the components. In fact, S_G is the void fraction, α, and S_O+S_W is the liquid hold-up,(1-α). Eq. (8) can alternatively be written as

$$K_M^T = \alpha K_G^T + (1-\alpha) K_L^T \tag{9}$$

If the liquid phase contains both water and oil, its isothermal compressibility can be expressed as

$$K_L^T = \beta K_W^T + (1-\beta) K_O^T \tag{10}$$

Combination of Eqs. (10) and (9) gives the isothermal compressibility of a gas/oil/water mixture as

$$K_M^T = \alpha K_G^T + (1-\alpha)\left[\beta K_W^T + (1-\beta) K_O^T\right] \tag{11}$$

The ratio of heat capacities of a single phase is given as

$$\gamma = \frac{C_p}{C_v} \tag{12}$$

For a gas/oil/water mixture, the heat capacities may be expressed as

$$\begin{aligned} C_p &= xC_{pG} + (1-x)C_{pL} \\ &= xC_{pG} + (1-x)\left[yC_{pW} + (1-y)C_{pO}\right] \end{aligned} \tag{13}$$

and

$$\begin{aligned} C_v &= xC_{vG} + (1-x)C_{vL} \\ &= xC_{vG} + (1-x)\left[yC_{vW} + (1-y)C_{vO}\right] \end{aligned} \tag{14}$$

Where x and y are the gas-liquid mass fraction and water-oil mass fraction, respectively.

It should be noted that mass fraction instead of void fraction is employed in these equations because the ratio of specific heats is a property based on unit mass.

Sound speed in a gas/oil/water mixture can be derived by substitution of Eqs. (7), (11), (13) and (14) into Eq.(4)

$$c=\left\{\frac{\left(xC_{pG}+(1-x)[yC_{pW}+(1-y)C_{pO}]\right)/\left(xC_{vG}+(1-x)[yC_{vW}+(1-y)C_{vO}]\right)}{\left(\alpha\rho_G+(1-\alpha)[\beta\rho_w+(1-\beta)\rho_O]\right)\left(\alpha K_G^T+(1-\alpha)[\beta K_W^T+(1-\beta)K_O^T]\right)}\right\}^{1/2} \tag{15}$$

RESULTS AND DISCUSSION

To estimate sonic velocity of a multiphase mixture using Eq. (15), four properties for each constituents need to be known, that is, density, isothermal compressibility, isobaric and isochoric heat capacities. These parameters are usually functions of pressure and temperature.

Properties of a gas mixture: ρ_G, C_{pG}, C_{vG} and K_G^T are thermodynamic properties of a gas mixture and can be calculated from its composition by using relevant theories (Parlaktuna and Gudmundsson, 1991).

Properties of water: Rowe and Chou (1970) developed an interpolation formula for brine to calculate ρ_W and K_W^T for different pressure, temperature, and salt concentration. The empirical relation is valid for $T<350$ °F (175°C), $p<345$ bara and sodium chloride concentrations less than 25 grams per 100 grams of solution. The isobaric heat capacity, C_{pW} of brine can be estimated by a correlation by Michaelides (1981). In the correlation, C_{pW} is related to temperature, pressure and salt concentration.The isochoric heat capacity, C_{vW} of water is usually difficult to measure. However, thermodynamics shows that the magnitude of isochoric heat capacity is nearly identical to isobaric heat capacity for a liquid. This argument has been used in this study.

Properties of oil: Wang *et al.* (1990) measured sound speed in various oils at different pressures and temperatures. An interpolation of the measured data has been used to estimate sound speed of an oil as a function of p, T and its API gravity. C_{vO} has been taken to be equal to C_{pO}.

The calculated values of the sonic velocities in an air/water system according to Eq. (15) are shown in Figure 1, where the sonic velocity is plotted against void fraction. The sonic velocity is dramatically lowered by the existence of a second phase, especially the admixture of a small-volume fraction of air in water. At 1 bara pressure the sonic velocity of an air-water mixture with void fraction 10^{-4} is lowered from 1460 m/s in pure water, to about 900 m/s. This character can be used to meter void fraction of a gas-liquid mixture by measuring the sonic velocity (Gudmundsson and Dong, 1992). At 1 bara pressure a minimum value of the sonic velocity is attained around void fraction 0.5. The sonic velocity is fairly constant for the mixtures with medium range of void fraction at low pressure.

The sonic velocity of the mixture is highly pressure-dependant. The sonic velocity of the mixture with void fraction 0.5 is about 20 m/s at 1 bara, but 350 m/s at 300 bara. The variation of the sonic velocity with void fraction becomes more gradual as pressure increases. The minimum point of the sonic velocity is shifted to larger values of void fraction.

Sonic velocities calculated from the model are compared with measured data in air-water systems reported by Karplus (1958), Henry *et al.* (1971), von Böckh and Chawla (1974), and Gudmundsson and Dong (1992). The details of the experimental conditions are listed in Table 1. The comparison of the calculation and the measured data by Gudmundsson and Dong (1992) is shown in Figure 2. It is observed that they agree well. This is also the case for the measured data by von Böckh and Chawla (1974) and Karplus (1958), as shown in Figure 3. Since the calculation is based on an isentropic process, it indicates the sonic wave propagation in air-water system is an isentropic process.

The comparison for the calculated sonic velocities and the measured data in a vertical air-water pipe by Henry *et al.*(1971) is shown in Figure 4. It is observed that the calculated values are smaller than the measured data when the void fraction is greater than 0.02. The discrepancy is probably resulted from the way the sonic velocities were obtained by Henry *et al.* (1971). Sonic waves usually consist of many components with different frequencies. It is well known that the higher the frequency the faster the moving speed. In consequence, the wave front mainly consists of high frequency components, the wave peak of low frequency components, and those in between of medium frequency components. Vongvuthipornchai (1982) clearly demonstrated this phenomena by comparing measured sonic velocities obtained using three different methods, as shown in Figure 5. Henry *et al.* (1971) obtained the sonic velocities by measuring the time difference of the wave fronts at two measuring stations. This gives the sonic velocities greater than those derived using intersection or peak method.

The comparison of the calculated values and the measured data by Vongvuthipornchai (1982) is shown in Figure 6. The measurement was carried out in an air-kerosene system, and the details of the experimental conditions are listed in Table 1. Since the pressure and temperature of the mixture varied in the experiment, the calculation shows an area embodied by two lines in the figure. It is observed that the measured sonic velocities are slightly higher than the calculated values. The reason is not quite clear. However, it indicates that the process must be isentropic; otherwise, the measured data should be lower than the calculated values since the latter is based on the isentropic processes.

The sonic velocity of a water-oil system is calculated and shown in Figure 7. The oil with the specific gravity of 0.82 is taken from a well of Norwegian North Sea. Unlike in a gas-liquid system the sonic velocity varies gradually with the water holdup, and it is much less pressure-dependent. This is because a liquid-liquid mixture has a density and compressibility which do not differ very much from those of its constituent liquids.

CONCLUSIONS

(1) A model for sonic velocity in multiphase mixtures has been presented. The calculated sonic velocity is in good agreement with measured data. It indicates that the process of propagation of sonic waves in a gas/liquid is isentropic.

(2) The sonic velocity of gas-liquid mixtures is strongly pressure-dependent. It is dramatically lowered by the presence of a small amount of gas, and this character can be used for two-phase flow metering purpose. The variation of the sonic velocity of gas-liquid mixtures becomes more gradual as pressure increases.

(3) The sonic velocity of oil-water mixtures increases monotonically from the sound speed of the pure oil to that of the pure water.

NOMENCLATURE

c	Sound speed, m/s
p	Pressure, bar
x	Gas/liquid mass fraction
y	water/oil mass fraction

α	Void fraction
β	Water/oil volumetric fraction
γ	Ration of specific heats
ρ	Density, kg/m^3

Superscripts and subscripts

S	Isentropic process
T	Isothermal process
G	Gas
L	liquid
M	Mixture
O	Oil
p	Isobaric process
v	Isochoric process
W	Water

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Table 1 List of Experimental Conditions in Test Sections

Authors	Pipe I.D. mm	Pipe L. m	Pipe Orientation	P bara	T °C
Gudmundsson & Dong, 1992	42.6	10	Horizontal	~ 1.4	21-25
Karplus, 1958	50.8	~ 1	Vertical	~ 1	20
Henry et al, 1971	52.6	~ 2	Vertical	1.7-4.4	26.7
von Böckh & Chawla, 1977	30.0	~ 3	Horizontal	~ 1	21
Vongvythiporn -chai, 1982	76.2	84	Horizontal	2.5-5.2	28-50

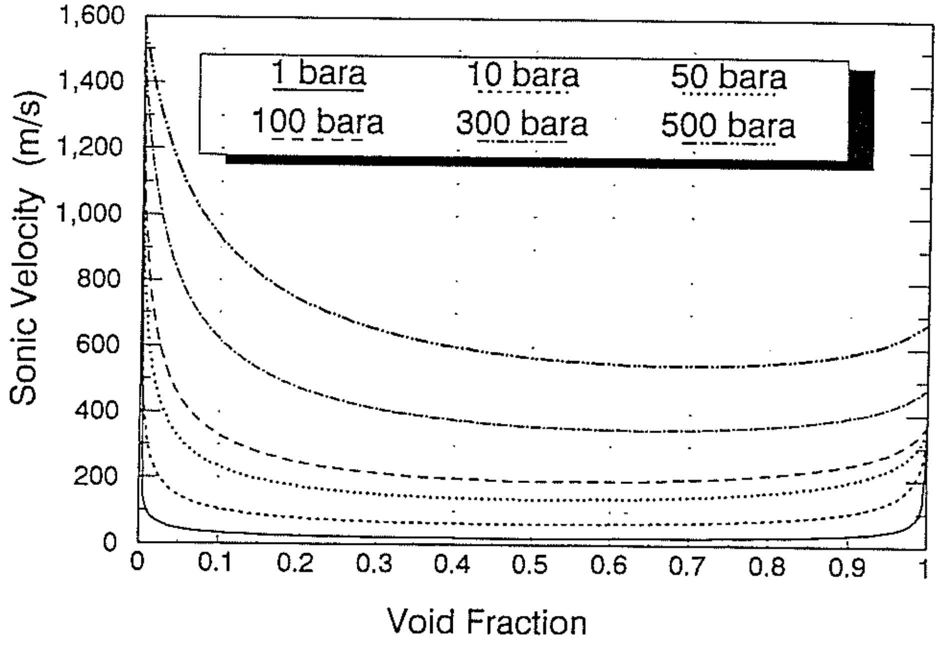

Figure 1 Calculated Sonic Velocities vs. Void Fraction and Pressure for Air-Water System at 25°C

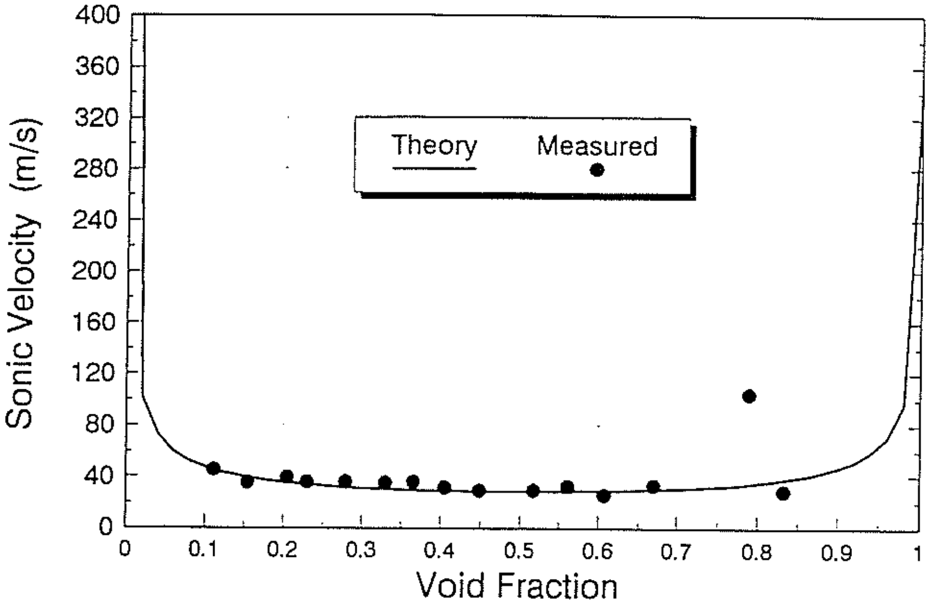

Figure 2 Comparison of Calculated Sonic Velocities with Measured Data in a Horizontal Air-Water Pipeline by Gudmundsson and Dong (1992)

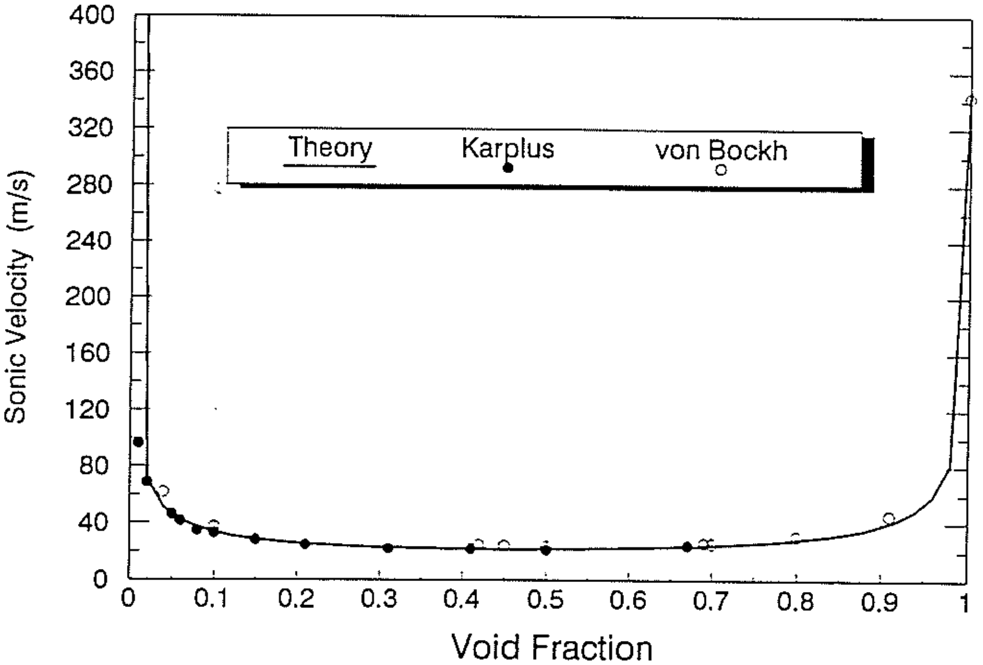

Figure 3 Comparison of Calculated Sonic Velocities with Measured Data in a Air-Water System by Karplus (1958) and von Bockh and Chawla (1977)

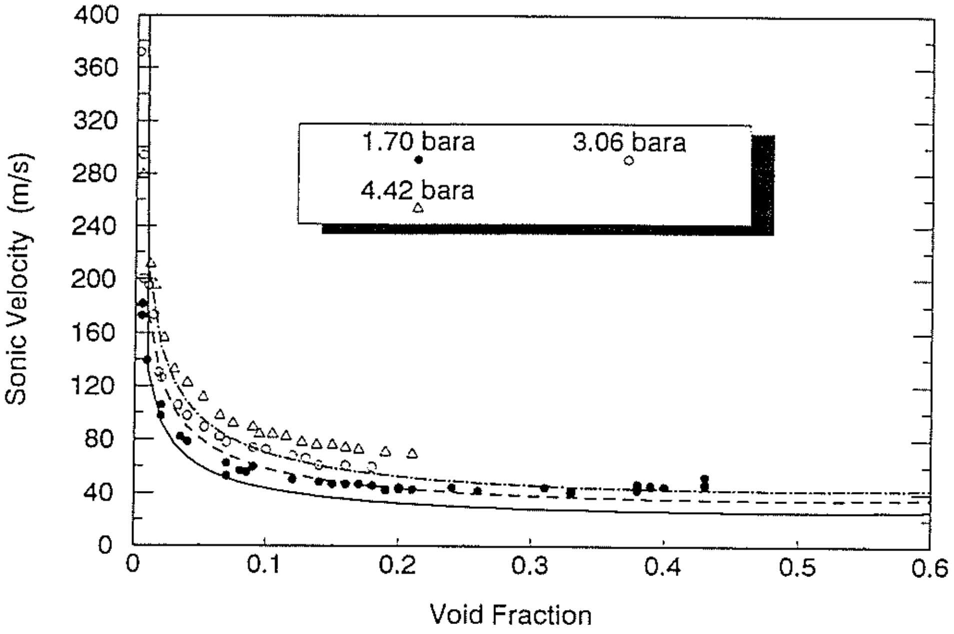

Figure 4 Comparison of Calculated Sonic Velocities with Measured Data in a Vertical Air-Water Pipe by Henry et al. (1971)

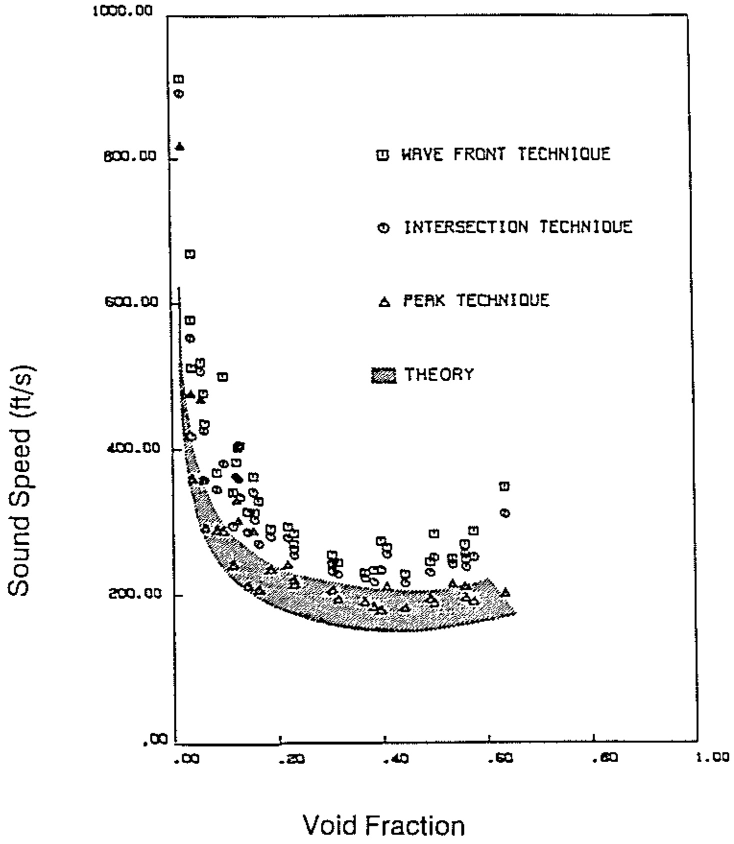

Figure 5 Sound Speed Measured by Three Techniques vs. Void Fraction

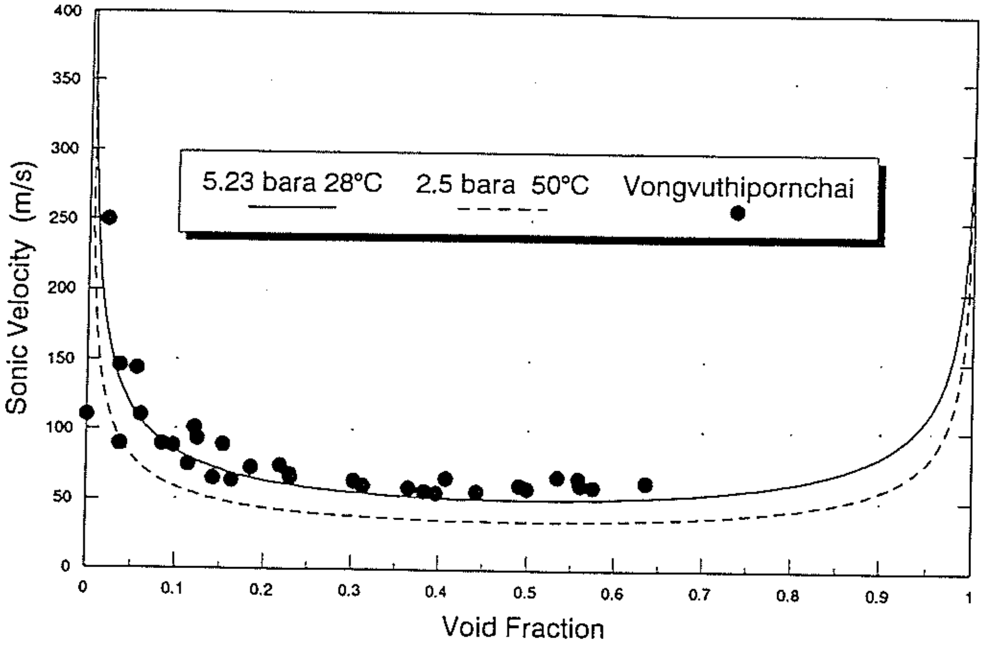

Figure 6 Comparison of the Calculated Sonic Velocities with Measured Data in a Horizontal Air-Kerosene Pipeline by Vongvuthiparnchai (1982)

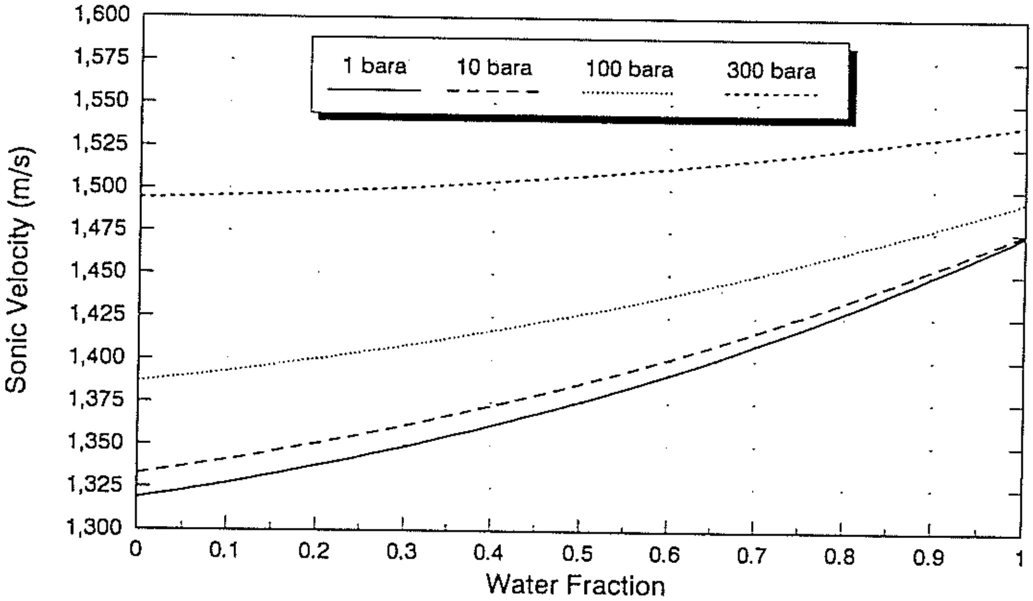

Figure 7 Calculated Sonic Velocity of an Oil-water Mixture vs. Water Fraction and Pressure at 25°C